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Mathematical Models for Insulating Packages and Insulating Packaging Solutions

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Mathematical Models for Insulating Packages

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MATHEMATICAL MODELS FOR INSULATING PACKAGES AND INSULATING PACKAGING SOLUTIONS

by

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I still remember the clear blue sky and lively squirrels on the trees which I saw out of my window on Goodway Ln, Memphis. I also can recall evening classes in the Mechanical Science Building and the huge parking lots on Central Ave. During my studying and working at the University of Memphis, there have been a lot of people who have supported my efforts in earning my doctoral degree.

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ABSTRACT

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Insulating packaging is used in a variety of applications to protect temperature-sensitive products from thermal damage. Several factors affect the performance of insulating packages. These factors include (i) heat transfer through the packaging material (conduction, convection and radiation), (ii) the configuration of the insulating package, and (iii) the choice of phase change materials. In this research, a comprehensive model, including all of these factors, was developed to effectively design and analyze the performance of the insulating package.

Nine one-dimensional heat transfer models, each consider only conduction, are reviewed and explored as candidates to be used for predicting the performance of insulating packages. Comparing calculated results from various models with experimental data, one spherical shell model (with the inside volume of the spherical shell equaling the inside volume of the rectangular container and the thickness of spherical shell equaling the thickness of the rectangular container) is identified most suitable for use in the following research.

Several versions of the spherical shell model which respectively consider conduction only, both conduction and external convection, multi-layered wall, and finally external convection and radiation and enclosed radiation together, were developed. Relationships among wall thickness, inside radius, package duration, ambient temperature, convection coefficients, emissivity of insulated material surface and product size are developed and discussed. General conclusions are reached as to these variables and input parameters.

The spherical shell model offers a unique basis for packaging analysis and design

due to the closed-form solutions of heat transfer through this spherical shell configuration. Based on this model, the insulating packaging solution is achieved by using ‘minimum packaging cost’ as an optimization target. This optimized design on the spherical shell model can be then transformed to rectangular configurations by using the same rule used for transforming rectangular configurations to the spherical shell configuration.

The comparison of results predicted from the present model with benchmark experimental data shows satisfactory agreement in terms of package duration. As a result, this research work offers not only an effective mathematical model for insulating package system, but also a unique transformation between the spherical and rectangular configurations, providing a straightforward tool, validated by benchmark experiments, for making insulating package design for the packaging industry.

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NOMENCLATURE

L = length of rectangular container

W = width of rectangular container

H = height of rectangular container

Δx = thickness of rectangular container or spherical shell

r = radius of spherical shell

k = conductivity of the insulated material

T = temperature

t = duration of the package

ρ = density of phase change material

ΔH = latent heat of phase change material

m = mass of phase change material

A = surface area

h_a = average convection heat transfer coefficient

ε = emissivity of surface

D = diameter of spherical shell

Nu = average Nusselt number

Ra = Raleigh number

P_r = Prandtl number

g = acceleration of gravity

γ = viscosity of air

β = volumetric coefficient of expansion of the air

σ = Stefan-Boltzmann constant

CHAPTER 1. INTRODUCTION

A "thermal insulating package" is a special package designed to concentrate on protecting temperature-sensitive products during shipping. An insulating package can maintain product temperatures within acceptable ranges and slow the deterioration of product in the distribution environment until it reaches the consumer (Singh, 2008).

A temperature-sensitive product is one whose characteristics are easily changed as the ambient temperature changes. Many products are temperature sensitive, including pharmaceuticals, food, biological materials, horticultural products, and even some industrial products. For best preservation, different products should remain in different temperature ranges. Fresh fruits and vegetables must be kept reasonably cold for best retention of food value and appearance. Seafood, meat and poultry require a storage temperature near 0℃ or even lower to maintain a longer shelf life in the retail stores. Cut flowers need temperatures between 4℃ and 12℃. Pharmaceuticals such as vaccines need to be kept within very restricted temperature ranges during shipping. Some pharmaceutical and biotech materials must be kept at room temperature (13-29℃). Others require refrigeration but must not be frozen. Almost all drugs break down at temperatures over 40℃, and nearly all biologically active drugs lose efficacy after being frozen or left at room temperature. For example, certain forms of artificial skin tissue must be maintained at temperatures near the normal body temperature. Other types of tissue must be stored between -30 and -50℃ (Cook, 1999; Singh, 2008). Insulating packaging should be considered as important as any other single component in the protection of medical and food products. Without proper protection during transportation, even the best and most expensive medicines and foods will lost effectiveness and could potentially become dangerous or lethal.

The distribution environments of such different products vary widely from case to case, location to location, industry to industry. From its manufacturing point to its final destination, a product may be exposed to quite vary shipping conditions. For example, a vaccine is manufactured in Puerto Rico and is sent via ground transport to the airport in San Juan. After spending time at a staging area, the shipment is loaded into the cargo hold of a transcontinental aircraft and flown to a distribution point in Canada. The manufacturer ships the carefully packed vials in bulk. In turn, the distributor repackages the medication in smaller quantities—with different packaging—and ships it to various clinics for use. The following conditions may impinge upon the product at various stages during this entire process:

- Puerto Rico: 32-35℃, <90% humidity
- Aircraft hold cargo: 35,000-ft altitude, possibly -6℃
- Final destination: subfreezing temperatures.

Both inside and outside the shipping container, extreme weather systems are at work. In general, no matter what environments the product-containers have been exposed to, three temperature ranges must be considered for product safety, depending on the products characteristics:

- Keep it frozen: below -7℃
- Keep it cool: -7~13℃
- Keep it warm: over 13℃.

Normally, temperature-sensitive products should be delivered with strict temperature control. From the point of manufacturer to the destination warehouse, temperature control is always maintained under the "cold supply chain" by using refrigerated or freezer trucks when distribution quantity is large. But when products are repacked and delivered to retail stores or customers, they are mixed with other products and the "cold chain" may be broken. This is commonly called the "last one mil" issue. In this case, thermal insulating package is used to complete the cold chain.

In the past, most efforts to provide a temperature-controlled supply chain have relied on 'passive' technology, using materials such as expanded polystyrene form (EPS) as insulators to keep temperatures down. Recently, however, there has been a move toward 'active' systems. New, active temperature-control systems use coolants such as dry ice and gel-packs as temperature-controlling mediums as there are relatively cost-effective and widely available worldwide (Robert Kayum, 2002). The temperature-controlling medium is normally Phase Change Material (PCM), which changes the physical state of the material from solid to liquid (or liquid to gas) and requires the addition of heat, which is used to balance the heat absorption when a shipped package experiences a hot environment.

In general, distribution and marketing of temperature-sensitive goods can be achieved via three different modes (Singh, 2008):

- Carrier-controlled thermal chains providing refrigerated trailers for the transportation of goods over longer distances;
- One-way systems offering the advantages of rapid package design and validation using various insulated shipping containers and PCMs; and
- Two-way systems, using reusable shipping containers, such as vacuum insulation panel (VIP), typically having impact-resistant exteriors and offering improved temperature control.

This dissertation concentrates on the one-way system. The one-way system has emerged as the most popular because of its ease of application. Insulated containers provide required insulation using material combinations and refrigerants differing in kind and quantity to maintain desired temperatures and preserve product quality. The most common configuration of insulating package is shown in Figure 1**.**

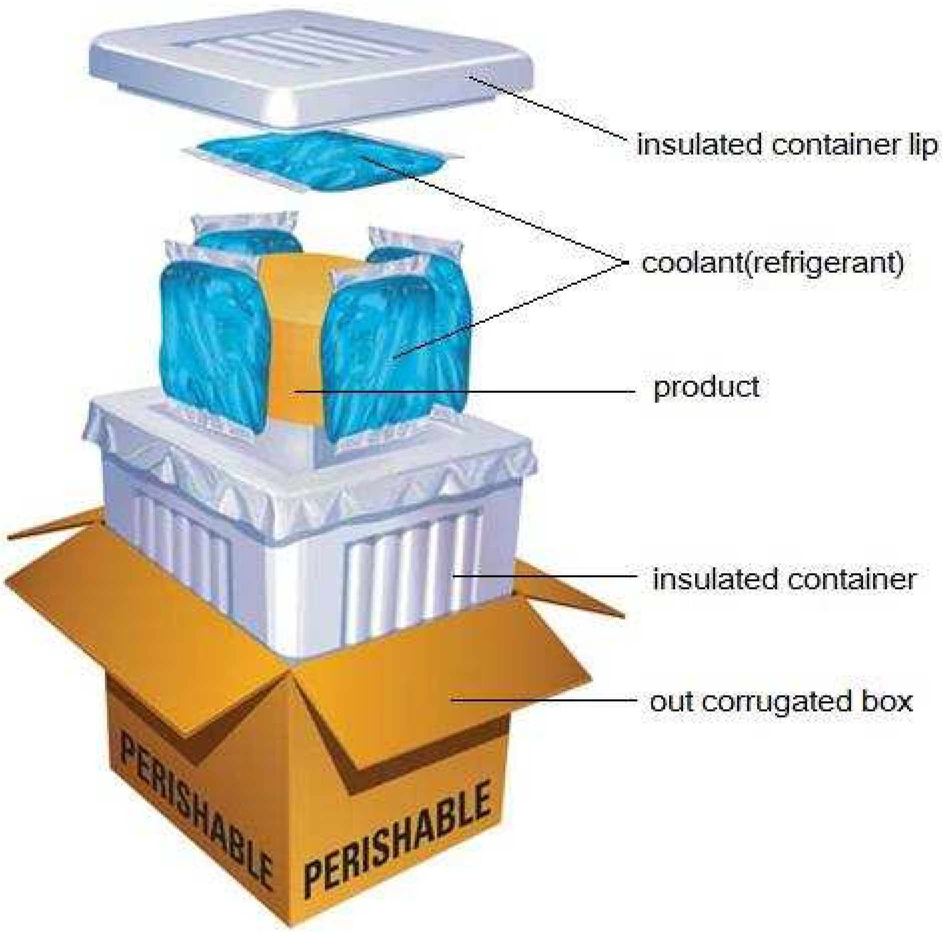


Figure 1. A configuration of insulating package

All forms of insulating packages focus on reducing the heat transfer from outside-to-inside or vice versa to maintain product temperature within acceptable ranges. Three heat transfer modes- conduction, convection and radiation- are involved in heat transfer through an insulating package. To reduce conduction heat transfer, the main method is to choose a proper insulation material offering a lower conductivity (or high thermal resistance, called R-value) and a certain thickness (such as 2-inch insulating medium). A list of thermal conductivities of commonly used insulation materials in packaging is shown in Table 1.

Table 1. Thermal conductivities of common used insulation materials (Choi, 2007)

Materials	Thermal Conductivity at $23\,°C$ ($w/m\cdot k$)
Air	0.026
Corrugated board	0.061
EPS foam	0.036-0.046
Polyurethane	0.031

The size, shape, thickness and structure of the insulating container wall directly affect the insulating capacity of the insulating package. Insulation materials utilize low thermal conductivity as a means of restricting the transfer of heat, although radiation and convection are also significant. Thin, multi–layered materials make better insulators than a single thick piece of material of the same thickness. Radiation can be restricted using a material with high reflectivity, such as aluminum foil. Most insulation materials absorb about 95% of the infrared radiation. But aluminum only absorbs 5% of infrared radiation. For this reason, the addition of aluminum foil can dramatically reduce infrared radiation, resulting in substantial improvement of the insulating ability of the package. Sealing the package also plays an important role in improving the insulating ability of the package. Air currents can flow in and out through very small openings and can carry enough heat to render even the best insulator ineffective. For example, corrugated boxes lined with EPS sheets on all six faces cannot insulate as well as a molded EPS container because there are always gaps along the edges where the EPS sheets meet (Choi, 2007).

Because of the variety of factors affecting the performance of the thermal insulating package, numerous researchers have worked on developing a comprehensive model which can represent these factors as precisely as possible, so as to predict the insulating capacity of packaging (Stavish,1984; Burgess,1999; Kositruangchai, 2003; Choi, 2007; Mo, 2008). All these works consider the existing forms of insulating packaging, and then use their data to predict the capability of other packaging.

The objective of this study is to develop an effective, comprehensive tool for making preliminary designs to produce prototypes of packaging for a given product in all the given conditions of protecting and preserving the product. Such designs and prototypes have traditionally been founded on experience, with much iteration through trial and error before getting the final packaging ready to use. This traditional process is time consuming and costly.

To accomplish the objectives set in this research, a new spherical shell model of the packaging container with multi-layered wall structure and all three heat transfer modes has been developed. Due to the one-dimensional feature of the model, analytical expression for all the involved parameters allows an optimization of packaging design before making actual packaging for test purposes.

A series of experiments has been conducted and experimental data are used in establishing this theoretical model.

CHAPTER 2. LITERATURE REVIEW

2.1 Insulation materials and insulating containers

Insulation materials are those with low thermal conductivities or high thermal resistances. Using these materials in insulating packaging conserves energy by reducing heat loss or gain, and reduces temperature variation within products.

The most common insulation material used in packaging is plastic foam, such as expanded polystyrene foam (EPS) and polyurethane foam (PU). Air trapped in EPS foam gives excellent insulating capacity, since air has very low heat conductivity. EPS foam is lightweight, inexpensive and stackable. The major drawbacks of EPS foam are its difficulty of disposal and difficulty in surface printing (Sasaki and Kato, 1999). In general, urethane offers a higher insulating capability compared with EPS foam. For this reason, urethane requires fewer refrigerants than does EPS foam. However, the cost of urethane foam is higher than that of EPS foam. The factors affecting thermal conductivity of cellular plastics are temperature and moisture. Absorbed moisture, depending on the temperature on either side of the insulation, is known to reduce thermal resistance of cellular plastics because it replaces the gas in the cellular structure (Desjarlais and Zarr, 2002).

The most common kinds of insulated container are made from EPS foam (Figure 2), molded polyurethane, vacuum insulation panels, gas-filled panels, and corrugated panels with liners or blankets.

Figure 2. Expanded polystyrene (EPS) foam container

Vacuum insulated panel (VIP) containers are constructed by assembling five vacuum insulated panels for the base and using a sixth panel for the lid. The base panels are often taped together to eliminate air gaps (Jenevieve, 2002).

The advantages of the VIP container are its high R-value, low shipping weight, and reusability. But the effectiveness of a VIP container is dramatically reduced once the vacuum is lost in any of the panels. In order to protect the VIPs, the manufacturers will often surround the panels with a buffer material such as EPS panels or flexible foam to protect the VIPs from potential damage.

Gas-filled panels (GFPs) are also used for insulating packages. These are composed of exterior films made from high density polyethylene (HDPE) and interior metalized films with low-emissivity surfaces. Low-conductivity, gas-filled cavities and a series of low-emissivity, honeycomb-type interior-film layers minimize heat transfer from radiation, convection and conduction. GFPs are essentially hermetic plastic bags that can take on a variety of shapes and sizes. Argon- gas- filling provides an effective thermal resistance level of R-7 per inch, krypton gas provides R-12.5 per inch, and xenon gas provides R-20 per inch.

Figure 3. Gas-filled panels container

2.2 Phase change materials (PCMs)

A variety of different refrigerants may be used in the insulated shipper. These products, often called temperature stabilizers or PCMs, can be formulated and conditioned at specific temperatures and can be used in concert with the product itself and the rest of the packaging to create an environment within the shipping container to thermally protect the product. They are made of non-toxic, food-grade, FDA-approved ingredients which are sealed in durable, leak-proof packaging. There are broad selections of materials available to maintain the temperature within narrow ranges between -50 ℃and 30 ℃ (Singh, 2008).

Figure.4 Gel-packs

Dry ice is solid carbon dioxide (CO_2) that sublimates directly to gaseous form.

During sublimation it maintains a temperature of -78℃ and absorbs heat with an exceptionally high latent heat of fusion because of its double phase change characteristic. However, some products cannot endure a temperature drop below -40℃. Also, dry ice cannot be used in an airtight container due to the expansion of particles as the solid sublimates to gas (one pound of dry ice can expand to over 8 cubic feet of carbon dioxide gas).

At 0℃, wet ice is safe to use for most refrigerated shipments with low cost, ready availability and the advantage of leakage. Gel packs (Figure.4) and form bricks are other commonly used refrigerants. A well designed container can utilize a combination of gel packs or form bricks to hold the product within a specific temperature range during transit. The availability of a variety of types of gel pack and form brick can be advantages in designing effective packing configurations.

Thermal energy storage in general and PCMs in particular have been a main topic in research for the last 20 years. This work falls within an area of international interest as it deals with energy saving, with the efficient and rational use of available resources, and with the optimum use of renewable energies. Applications of PCMs are found in cogeneration equipment or in installations with reduced prices for electrical energy which is consumed during off-peak hours. Thermal inertia and thermal protection present another area where PCMs have gained a higher percentage of the market (Belen Zalba, 2003). Application of PCMs in packaging is a type of thermal protection.

When PCMs are used in packaging, melting points and latent heat of PCMs are two properties which concern packaging designers. The main analytic techniques used to study phase changes are conventional calorimetry, differential scanning calorimetry (DSC) and differential thermal analysis (DTA). Yinping Zhang (1999) reviews the above-mentioned conventional methods of PCMs property analysis and points out their limitations, proposing a simple method for determining phase change temperature under cooling, enthalpy, and thermal conductivity in solid and liquid phases. Temperature graphs are drawn and properties evaluated for comparison with the graphs of other known materials (typically pure water) used as reference. S. P. Singh (2007) gives another method for determining melting points and latent heat of PCMs, called the ice-melting test. The principle of this method is heat balance, the principle that the heat lost is equal to the heat gained by PCMs.

2.3 Distribution environments and their simulation

For shipping temperature-sensitive products, the environment of distribution, especially temperature, should be known clearly. During shipping, products pass through a variety of distribution processes as they are delivered from manufacturer to customer. During these processes, the product is exposed to several thermal cycles. ISTA 7D “Thermal Controlled Transport Packaging for Parcel Delivery System Shipment” provides generic cyclic temperature-controlled transport packages. These cyclic profiles are not meant to represent the actual transit environment for any temperature-controlled shipping application, but are intended only as an introduction to the thermal testing process. For any particular distribution system, especially in regulated industries such as high-value pharmaceuticals, temperature profiles must be developed from data taken in actual shipping environments (Kazuhisa, 2007). The winter and summer profiles from ISTA 7D are ‘cold shipping and cold receiving’ and ‘hot shipping and hot receiving’, respectively.

2.4 Mathematical models for insulating packages

Most investigations focus on the expression of R-value in packaging system. A more practical and simple procedure to determine R-value is proposed by Burgess (Burgess, 1999). This procedure, the so called ‘ice-melt test’, is based on the principle that 1 lb of ice at its melting point requires 144 BTU of latent heat to melt. In this method, an insulating package with a known quantity of ice inside is stored for a designated time period in a constant temperature environment. By comparing the remaining quantity of ice to the original quantity, the ice melt-rate is determined. Using this rate, the rate of heat flow can be obtained and subsequently the thermal resistance can be determined. This procedure requires no special equipment or temperature sensors, but is still a very effective method for determining the insulating ability of a package. In this case, the R-value on the thermal resistance of packages is the reciprocal of the thermal conductivity, calculated by the following equation (Burgess, 1999):

$$R-value=\frac{A\times\Delta T}{meltrate\times latentheat}=\frac{A\times\Delta T}{Q} \tag{2-1}$$

where R-value is the thermal resistance of the container wall in $ft^2\cdot{}^\circ F\cdot hr/BTU$, A is the inside surface area of the package in ft^2, ΔT is the temperature difference between outside air and refrigerant used in ${}^\circ F$, and Q is the heat transfer rate in BTU/lb. The melt rate is the rate that ice melts per unit time and is equal to the weight of ice melted divided by the melting time (lb/hr). When regular ice is used, the latent heat of ice is 144 BTU/lb.

A mathematic model is very useful in predicting efficient insulating packages. Proper simulation models can significantly reduce time-consuming efforts of the preliminary specifications, fabrication and subsequent validation tests of a packaging.

It also eliminates over-packaging and the resultant unnecessary costs. However, there have been very few attempts made to predict the capability of insulation packages (Stavish, 1984; Burgess, 1999; Kositruangchai, 2003; Choi, 2007). Stavish (1984) proposes a simple model to predict the heat flow through the package using the concept of total thermal resistance. The total thermal resistance, R_t , is the summation of R-values of the material and the air film resistances on the inside and outside of the insulating packaging. This can be expressed as

$$R_t = R_i + R_{meterial} + R_o \tag{2-2}$$

where R_i is the air film resistance on the inside surface of the package, $R_{material}$ is the thermal resistance of the insulation material and R_o is the air film resistance of the outside surface of the package.

Burgess (1999) also proposes a model for predicting the R-values of the insulating package. In his report, the system R-value could be predicted using following fitted equation

$$R_{system} = 3.9th + 1.5np + 3.2nf \qquad \text{(±20\% accuracy)} \tag{2-3}$$

where th is the average wall thickness in inches, np is the number of plain surfaces, and nf is the number of aluminum foil surfaces. In this equation, the system R-value is split into three parts: the effects of conduction, convection and radiation.

Seung-jin Choi (2004) developed a model which includes the heat transfer through the packaging material (conduction, convection and radiation), the geometry of the insulating package and the contact resistance between the product and package to predict the performance of the insulating package. In his study, the system R-value includes the resistance to heat flow of the product in addition to that provided by the wall. The system R-value in $m^2 \cdot K/W$ is

$$R_{system} = 0.27th + 0.26np + 0.56nf \tag{2-4}$$

where th is the average wall thickness in cm, np is the number of plain (non-foil) air to material surfaces, and nf is the number of aluminum foil surfaces. The major differences between the two models (given in equations 2-3 and 2-4) come from consideration of two important factors: geometric effects and contacts between the product and the insulating package.

However, all these researchers supposed that the insulating container existed. Based on this supposition, the investigators developed the expressions of insulating package system R-value for determining heat transfer on a one-dimensional plane surface and subsequently for predicting the quantity of PCMs or the allowable shipping time which the insulating package system can provide under specific ambient temperature and desired product desired temperature.

Unfortunately, before shipping a temperature-sensitive product, neither the interrelated dimension of an insulated container nor the quantity of PCMs is known. All of these parameters of the insulating package system need to be determined by the designer. As a result, an efficient and user-friendly model is desirable for designers to determine all these parameters of a packaging system for all given constraints.

2.5 Insulating package design and testing

Designing a shipping container is not a simple task. Of concern are the allowable interior environmental conditions within the container, the expected conditions outside the container, the mass of the product, the mode of transportation, the duration of the shipping chain, and the number of times that the product will change hands. Testing a container and product under simulated conditions provides information about a package's ability to maintain the temperature of an article within the required range (Taborsky, 1999).

Mathematical models are very useful in thermal packaging design. An effective mathematical model can be of significant value not only for insulated containers (materials, dimensions and structure), but also for phase change materials (type and quantity). Finite element analysis can be used on insulating package designs. In this analysis, designers consider the entire load—payload, coolant, insulation—and break it up into very small elements. One should rate each material as to its material properties, such as conductivity and heat capacity. Then a transient thermal analysis can be done. Validation of the prediction should be performed.

By using ASTM or ISAT standards and a temperature profile in shipping environment, a testing temperature profile can be developed. By selecting containers, calculating the amount of PCMs, putting the temperature monitors (recorders) into thermal packaging system, the prototype testing can be started. After analyzing the test data, the design can be finalized if the specification of objectives is achieved. Otherwise, the process is started over again.

CHAPTER 3. THEORETICAL DEVELOPMENT AND EXPERIMENT VERIFICATION

3.1 International System of Units (SI)

The metric system is an international decimalized system of measurement common to most of the world. The International System of Units (SI) is the current international standard metric system, also called meter-kilogram-second system. It is employs the meter, kilogram, second, ampere, Kelvin, candela and mole as basic units. In this dissertation, SI units are used for all physical quantities discussed here.

3.2 Basic theories of heat transfer

Heat is the form of energy that can be transferred from one system to another as a result of temperature differences. The science dealing with the determination of rates of such energy transfer is heat transfer. The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature. Heat can be transferred in three different modes: conduction, convection and radiation.

3.2.1 Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between particles. Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to collisions and diffusion of molecules during their random motion. In solids, conduction is due to the combination of vibration of molecules in a lattice and energy transport by free electrons.

The rate of heat conduction through a medium depends on the geometry of the medium, its thickness, and the material of the medium, as well as on the temperature difference across the medium. Consider steady-heat conduction through a large plane wall of thickness Δx and area A, as shown in Figure 5. The temperature difference across the wall is $\Delta T = T_2 - T_1$ in ℃. Based on Fourier's law of heat conduction, the rate of heat conduction in W through a plane layer is

$$Q_{cond} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \tag{3-1}$$

where the constant of proportionality k is the thermal conductivity of the material, which is a measure of the ability of a material to conduct heat in $W/m \cdot °C$. Thermal conductivities of materials vary with temperature. In practical packaging applications, the change of temperature is not so large, so the thermal conductivity is regarded as constant. Thermal conductivities of insulation materials, such as EPS, vary with their density. Table 2 shows the data of thermal conductivities of EPS while the density of material changes (Yucel, 2007).

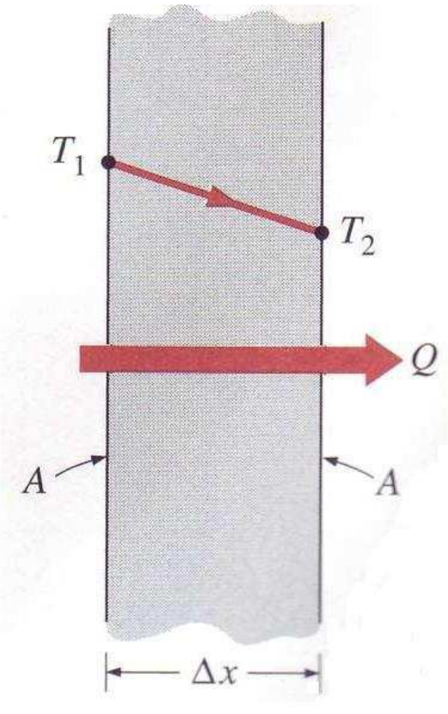


Figure 5 Heat conduction through a wall

Table 2. Thermal conductivities of EPS on various densities

Density of EPS(Kg/m^3)	Thermal conductivity($W/m\cdot°C$)
10	0.046
15	0.042
20	0.039
25	0.037
30	0.036

3.2.2 Convection

Convection，as discussed in this dissertation, is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfer. Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or wind. In contrast, convection is called natural convection if the fluid motion is caused by buoyancy forces that include density differences due to variations of temperature in the fluid.

Despite the complexity of convection, the rate of convection heat transfer in W is observed to be proportional to the temperature difference, and is conveniently expressed by Newton's law of cooling:

$$Q_{conv} = hA_s(T_s - T_\infty) \qquad (3\text{-}2)$$

where h is the convection heat transfer coefficient in $W/m\cdot°C$, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface.

3.2.3 Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of atoms or molecules. Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium. In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. Thermal radiation is the form of radiation emitted by bodies because of their temperature.

Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K or R) is given by the Stefan-Boltzmann law as

$$Q_{emit,\max} = \sigma A_s T_s^{\ 4}$$

where $\sigma = 5.670 \times 10^{-8} W / m^2 \cdot K^4$ is the Stefan-Boltzmann constant. The idealized surface that emits radiation at this maximum rate is called a black body. The radiation emitted by all real surfaces is less than the radiation emitted by a black body at the same temperature, and is expressed as

$$Q_{emit} = \varepsilon \sigma A_s T_s^{\ 4} \tag{3-3}$$

where ε is the emissivity of the surface. The property of emissivity, whose value is in the range $0 \le \varepsilon \le 1$, is a measure of how closely a surface approximates a black body for which $\varepsilon = 1$. The emissivity of some surfaces is given in Table 3.

Table 3. Emissivity of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Polished stainless steel	0.17
White paper	0.92-0.97
Asphalt pavement	0.85-0.93
Human skin	0.95
Water	0.96
Vegetation	0.92-0.96

3.3 Different models for conduction through the walls of insulating container

Real insulating containers have three-dimensional geometries. However, it is difficult to get a solution for a three-dimensional model. As a result, in the literature, one-dimensional plane wall conduction is popularly modeled for calculating the conduction heat transfer through the walls of an insulating container in packaging engineering. In this section, several different one-dimensional models are introduced and evaluated by comparing with experimental data collected in this research.

To simplify the basic models, approximating the real heat transfer through insulating package, it is assumed that:

1. Conduction is the main mode of heat transfer, since insulating containers mostly stand in covered trucks or warehouses. In this case, convection and radiation can be ignored in the initial approach;
2. A steady-state condition exists. The outside temperature is T_o; the inside temperature is T_i;

3. The phase change material (PCM) is the main thermal sink to keep the temperature of the product to remain constant or in a proper range before the PCM is completely melted. Normally latent heat of PCMs is higher than sensible heat. In insulating package, after PCM completely changes its phase to liquid, the inside temperature of the package increases quickly. In short, all the heat which transfers from the outside to the inside of the package is absorbed by the PCM;
4. When the phase change happens, the temperature of PCM in the package is uniformly constant at its melting point;
5. Duration of the package is the time that it takes for phase changing of the PCM in this dissertation research. Realistically, duration of the package is longer than this definition as the temperature of PCMs could be lower than the melting point and the allowance temperature of product could be higher than the melting point for some of products. In this case, sensible heat gain can be balanced through increasing the PCM temperature to provide for a longer shipping time.

Therefore, the result can be obtained,

$$Q_{cond} = \Delta H \times m / t \tag{3-4}$$

where Q_{cond} is the rate of heat conduction through the package enclosure in W, ΔH is the latent heat of PCMs in KJ / Kg, m is the mass of PCM in Kg, t is the time during which phase change (duration of package) is lasted in s.

3.3.1 Plane wall models

Considering the insulating container as a thin plane wall, the following equation can be established, via equation 3-1 and 3-4:

$$kA\frac{T_o - T_i}{\Delta x} = \Delta H \times m / t$$

Solving the equation, the PCMs melting time t can be obtained:

$$t = \frac{m \cdot \Delta H \cdot \Delta x}{k \cdot A \cdot (T_o - T_i)} \tag{3-5}$$

In the equation 3-5, the PCM melting time (packaging duration) t is in direct proportion to the amount of PCMs and the wall thickness of the container, and in the inverse ratio of temperature difference between the inside and the outside packaging and the surface area of the container.

In much of the literature, surface area is taken to be the inside surface area of the container (Choi, 2007; Singh, 2008). Seung-Jin Choi gives another suggestion using the geometric average of the inside and outside areas of the container instead of simply inside surface area, in equation 3-5. The Fourier Law of heat conduction states that, in terms of effective area $A(x)$,

$$Q = k \cdot A(x) \cdot \frac{dT}{dx} \tag{3-6}$$

In this method, the thickness of the container is divided as many very thin layers. The thicknesses of these thin layers are taken as a constant dx, in which x is the distance measured from outside wall toward inside wall of the container. meanwhile, the area of each layer can be found as a function of x. By integrating equation 3-6 to x, an expression for the effective area can be achieved. Directly take geometric and arithmetic averages for the inside and outside areas of the container offers two candidates for approximation of the effective area expression. Both geometric and arithmetic averages of the inside and the outside area for effective area have been well studied and concluded that the geometric average approach is equivalent to the integral method (Choi, 2007).

Therefore, one approximation for the effective area is simply taking the inside surface area of the container, that is:

$$A = A_i \tag{3-7}$$

The geometry average of the inside and outside areas of the container is another approximation for the effective area, that is:

$$A = \sqrt{A_i \cdot A_o} \tag{3-8}$$

In the following studies, efforts are devoted to the models presented in equation 3-7 and equation 3-8.

3.3.2 Calculate the rate of heat conduction by shape factors

When considering the various geometries of containers, there is an empirical method to calculate the rate of heat conduction by introducing a shape factor, S, in m. The effective area has been determined by graphical techniques from analytical solutions and from the results of electrical analogs for various geometries. Table 4 gives some of the encountered configurations used in this dissertation. When conduction shape factor S is known, Q_{cond} may be directly calculated from

$$Q_{cond} = k \cdot S_{tot} \cdot \Delta T \tag{3-9}$$

where ΔT is the overall temperature difference between two isothermal boundaries and k is the thermal conductivity of the material in $W/m \cdot °C$ (Karlekar, 1982).

For a rectangular container with inside dimensions of $L \times W \times H \times \Delta x$, the shape factors can be calculated as below:

For walls: $$S_w = \frac{L \times W}{\Delta x} + \frac{W \times H}{\Delta x} + \frac{H \times L}{\Delta x}$$

For edges: $$S_e = 0.54(L + W + H)$$

For one corner: $S_c = 0.15\Delta x$

The total shape factor is:

$$S_{tot} = 2S_w + 4S_e + 8S_c \tag{3-10}$$

Applying equation 3-9 to equation 3-4, it can be found that

$$t = \frac{m \cdot \Delta H}{k \cdot S_{tot} \cdot \Delta T} \tag{3-11}$$

3.3.3 Spherical shell models

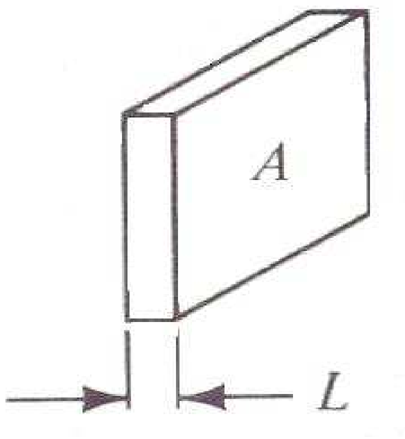
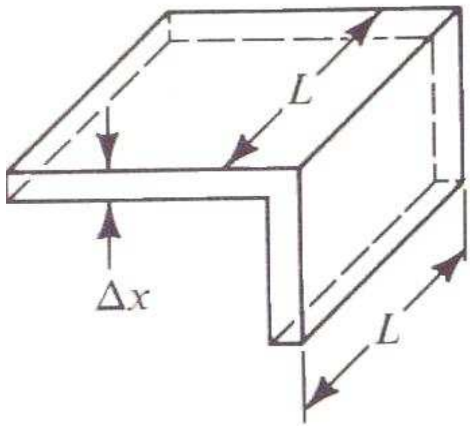
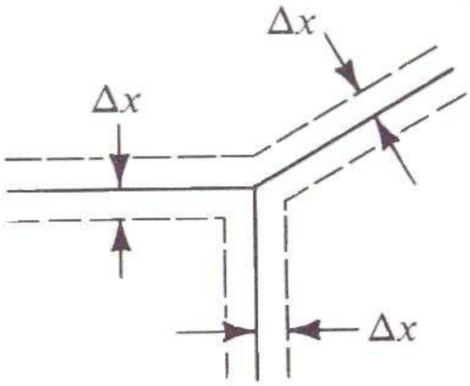
Normally, the insulating container is a rectangular box, and the dimensions of the container in length, width and height are in the same order. Therefore, it is rational to develop a spherical shell model which approximately simulates conductivity through a rectangular insulating container. This is an especially reasonable method for approximating a cubic container. To obtain an appropriate protection for a temperature sensitive product and minimize the volume (and cost) of the package, the desired configuration of the insulating package should be as sketched in Figure 6, with the product located in the center of the package and the coolant (PCMs) fully filling the space between the product and the container.

For a spherical shell, the rate of conduction heat transfer can be analytically expressed as (Karlekar, 1982):

$$Q_{cond} = \frac{4\pi \cdot r_o \cdot r_i \cdot k \cdot (T_i - T_o)}{r_o - r_i} \tag{3-12}$$

where r_i is the inside radius of spherical shell in m, r_o is the outside radius of spherical shell in m, T_i and T_o are the temperature over inside and outside surface in ℃, respectively.

Table 4. Conduction shape factors

Physical system	Schematic	shape factor
Plane wall		$\frac{A}{L}$
Conduction through the edge section of two walls-inner		$0.54L$
conduction through the corner section of three homogeneous walls		Δx

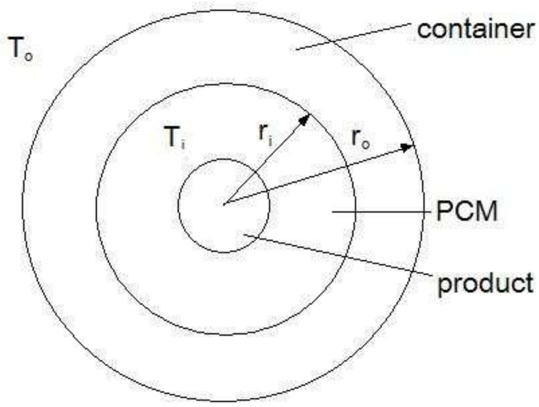


Figure 6. Spherical shell model

Applying the equation 3-12 in equation 3-4, the following expression can be developed:

$$t = \frac{m \cdot \Delta H \cdot (r_o - r_i)}{4\pi \cdot r_o \cdot r_i \cdot k \cdot \Delta T} \tag{3-13}$$

In this spherical shell model, it is necessary to find a correlation between the spherical dimensions in terms of inner radius and outer radius of the shell, and the actual rectangular container dimensions of length, width and height. To develop a reliable correlation, several selections have been applied. The final choice will be justified by comparing the theoretical results with the benchmark experimental data.

If the inside dimensions of an insulating container are $L \times W \times H \times \Delta x$, where the thickness of container is Δx, the choices of r_i and r_o can be expressed respectively as follows:

1. Both the inside and outside volumes of the spherical shells are respectively equal to the inside and outside volumes of the rectangular container.

$$r_i = \sqrt[3]{L \cdot W \cdot H / (\frac{4}{3}\pi)}$$

$$r_o = \sqrt[3]{(L + \Delta x) \cdot (W + \Delta x) \cdot (H + \Delta x) / (\frac{4}{3}\pi)}$$

2. The inside volume of the spherical shell equals the inside volume of the rectangular container and the thickness of spherical shell equals the thickness of the rectangular container.

$$r_i = \sqrt[3]{L \cdot W \cdot H / (\frac{4}{3}\pi)}$$

$$r_o = r_i + \Delta x$$

3. The outside volume of the spherical shell equals the outside volume of the rectangular container and the thickness of spherical shell equals the thickness of the rectangular container.

$$r_i = r_o - \Delta x$$

$$r_o = \sqrt[3]{(L + \Delta x) \cdot (W + \Delta x) \cdot (H + \Delta x) / (\frac{4}{3}\pi)}$$

4. Both the inside and outside surface areas of the spherical shell respectively equal the inside and outside surfaces of the rectangular container.

$$r_i = \sqrt{(L \cdot W + W \cdot H + H \cdot L) / (2\pi)}$$

$$r_o = \sqrt{[(L + \Delta x) \cdot (W + \Delta x) + (W + \Delta x) \cdot (H + \Delta x) + (H + \Delta x) \cdot (L + \Delta x)] / (2\pi)}$$

5. The inside surface area of spherical shell equals the inside surface area of the container and the thickness of spherical shell equals the thickness of container.

$$r_i = \sqrt{(L \cdot W + W \cdot H + H \cdot L) / (2\pi)}$$

$$r_o = r_i + \Delta x$$

6. The outside area of the spherical shell equals the outside area of the rectangular container and the thickness of the spherical shell has the thickness of the rectangular container.

$$r_i = r_o - \Delta x$$

$$r_o = \sqrt{[(L+\Delta x)\cdot(W+\Delta x)+(W+\Delta x)\cdot(H+\Delta x)+(H+\Delta x)\cdot(L+\Delta x)]/(2\pi)}$$

3.4 Experimental Set-up and validation of the proposed models

A total of nine correlations have been proposed above. The experiment was set up to identify the best formulation to be used in the simulation of the insulating package. The experiment conducted in this research takes the design as shown in Figure 6, but on rectangular containers.

In the experiments, all the insulated containers are made from EPS foam, which is commonly, used in commercial containers and readily available. An aluminum block is used to simulate the product. The temperature of the block is referred to as the product temperature and is taken with a temperature recorder. The PCM used in experiments is broken ice whose temperature is at 0℃ when initially loaded into the container. To set up the experiment properly, the broken ice has been tamped to make sure that it is tightly packed so that no inside air produces convection heat transfer. The insulating containers used in the experiments are shown in Table 5. Their thermal conductivity is calculated based on Table 2.

The experiment apparatus is summarized below:

1. Environment chamber

To accelerate the experiments and simulate only the most rigorous environment, the temperature inside the environment chamber is controlled as a constant. Figure 7 shows the environment chamber used in experiments.

Table 5. Insulating containers

No.	Inside dimension ($L \times W \times H$: mm)	Thickness (Δx : mm)	Density of EPS (kg/m^3)	Thermal conductivity ($W/m \cdot °C$)
1	210x145x145	50	30	0.036
2	200x152x140	28	16	0.042
3	225x187x260	55	26	0.037
4	298x213x280	26	25	0.037
5	204x155x108	38	22	0.038
6	204x155x155	38	25	0.037
7	208x158x150	35	19	0.040
8	370x290x217	72	34	0.035
9	230x155x172	40	19	0.040
10	217x172x165	60	18	0.040
11	170x150x196	56	25	0.037
12	230x155x192	40	20	0.039

2. Temperature sensor and recorder

The temperature sensors used in the experiments are from Sensitech Inc. Although each sensor was pre-calibrated by the supplier, a mercury thermometer is also used to recheck the sensor's default temperature before the experiments.

Two types of temperature sensors are used in the experiments: an Ambient Temperature Monitor (Figure 8) and a Probe Temperature Monitor (Figure 9). The ambient temperature monitor records the ambient temperature of the packaging, and the probe temperature monitor records the temperature of the product.

At the end of each experimental case, the data from the monitors are transferred to a computer via a USB Interface Reader (Figure 10). The monitors are then reconfigured by Temp Tale Manager Desktop 4.2 (Figure 11) to start a new experimental case.

Figure 7. Environment chamber

Figure 8. Ambient temperature monitor

Figure 9. Probe temperature monitor

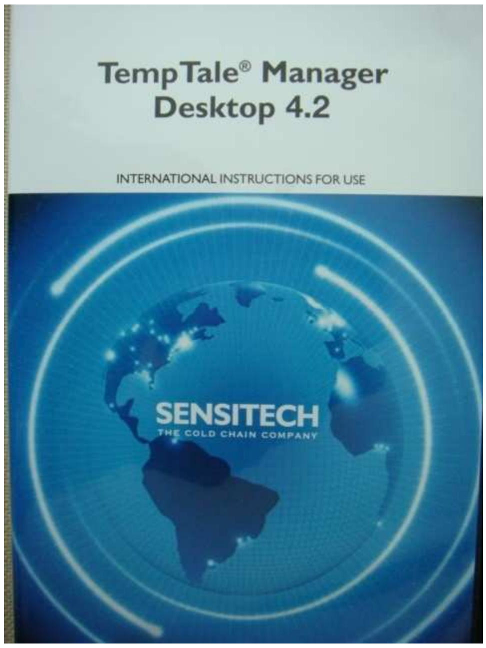


Figure 10. USB interface readers **Figure 11.** Temp-Tale manager desktop 4.2

Experiment procedure:

1. Turn on the power supply to the environment chamber to reach the constant temperature for which the experiment is designed.
2. Open the insulating container, weight and load broken ice into the container, and tamp it well. The filling of the broken ice at this step is to the level ready placing product block.
3. Load the product block with the probe temperature monitor at the center of the top of the tamped ice.
4. Weight enough broken ice to fully fill the container and again tamp it well.
5. Turn the probe temperature monitor on and use tape to seal the container completely.
6. Put the whole package into environment chamber and turn the ambient temperature monitor on.

7. Leave the package in chamber to ensure the ice melts completely.
8. The experimental case is now complete.
9. Read the temperature data via Temp Tale Manage Software and determine the ice melting time (i.e. the duration of packaging) from the recorded data.

The original data of the duration and the ambient temperature from the experiment, the calculated solutions for different mathematic models and errors defined as the difference between the calculated results and experimental data, can be found in Table 6.

Table 6. Experiment recorder and calculated solutions on different mathematic models

Container No.	Mass of ice (g)	Melting time (hours)	Ambient temperature (℃)	Model No. 1		Model No. 2		Model No. 3		Model No. 4		Model No. 5		Model No. 6		Model No. 7		Model No. 8		Model No. 9	
				Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)
1	1993	23	52.5	29.94	30.2	22.95	-0.2	22.21	-3.4	18.08	-21.4	25.28	9.9	35.27	53.3	15.96	20.82	20.82	9.5	26.48	15.1
1	1472	20	48.6	23.89	19.5	18.31	-8.4	17.73	-11.4	14.43	-27.9	20.17	0.9	28.14	40.7	12.74	-36.3	16.62	-16.9	21.13	5.7
2	1820	16.5	49	14.46	-12	12.33	-25.3	12.12	-26.5	9.68	-41.4	14.19	-14	17.24	4.5	8.56	-47.8	11.58	29.8	13.34	-19.1
2	2105	14	52.5	15.61	11.5	13.3	-4.9	13.09	-6.5	10.45	-25.4	15.32	9.4	18.62	3.3	9.25	-33.9	12.5	-10.7	14.41	2.9
2	2033	13.5	53.8	14.71	5.1	12.54	10.4	12.33	-11.9	9.84	-29.7	14.44	3.1	17.54	25.3	8.71	-37.8	11.78	-15.9	13.57	-3.1
2	2081	14.2	49.5	16.37	12	13.96	0.3	13.72	-2	10.95	-21.8	16.06	14.7	19.52	39.4	9.7	-30.7	13.11	-6.4	15.1	7.9
3	3819	32	46	38.49	20.3	30.85	-3.6	30.07	-6	24.1	-24.7	34.44	7.6	45.36	41.8	21.34	-33.1	28.36	-11.4	34.64	8.3
3	2979	21	50	27.64	31.6	22.92	9.2	22.46	6.9	17.94	-14.6	26.01	23.9	32.75	56	15.91	-24.2	21.32	1.5	25.21	20
4	8786	36	45	30.91	-14.1	28.11	-21.9	27.86	-22.6	22.07	-38.7	33.4	-7.2	37.47	4.1	19.51	-45.8	27.03	-24.9	29.4	-18.3
4	9265	34	49.2	29.81	-12.3	27.11	-20.3	26.87	-21	21.28	-37.4	32.22	-5.2	36.14	6.3	18.82	-44.6	26.07	-23.32	28.35	-16.6

Table 6. Experiment recorder and calculated solutions on different mathematic models (continued)

Container No.	Mass of ice (g)	Melting time (hours)	Ambient temperature (℃)	Model No. 1		Model No. 2		Model No. 3		Model No. 4		Model No. 5		Model No. 6		Model No. 7		Model No. 8		Model No. 9	
				Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)
4	9830	37	47	33.11	-10.5	30.11	-18.6	29.84	-19.4	23.64	-36.1	35.78	-3.3	40.14	8.5	20.9	-43.5	28.95	-21.7	31.49	-14.9
5	1695	18.3	48	23.34	27.3	18.65	1.8	18.17	-0.9	15.16	-17.3	21.3	16.2	27.91	52.3	13.06	-28.8	17.17	-6.3	20.9	14
5	1737	19	49.5	23.2	22	18.54	-2.43	18.06	-5	15.07	-21	21.17	11	27.91	46	12.98	-32	17.07	-10.2	20.78	9.3
6	3055	28	48	34.86	24.5	28.49	1.7	27.85	-0.5	22.22	-20.6	32.04	14.4	41.16	47	19.76	-29.4	26.36	-5.9	31.61	12.9
6	2803	24.8	52.6	24.72	-0.3	20.51	-17.3	20.09	-19	16.05	-35.3	23.27	-6.2	29.3	18.1	14.24	-42.6	19.07	-23.1	22.55	-9.1
7	2646	23	48	25.57	11.2	21.21	-7.8	20.78	-9.7	16.6	-27.8	24.06	4.6	30.3	31.7	14.72	-36	19.73	-14.2	23.32	1.4
7	2820	22	49.5	26.42	20	21.92	-0.4	21.47	-2.4	17.15	-22	24.87	13	31.31	42.3	15.21	-31	20.39	-7.3	24.1	9.5
7	2153	17	48.6	20.55	21	17.05	0.3	16.7	-2	13.34	-21.5	19.34	14	24.35	43.3	11.83	-30	15.86	-6.7	18.74	10.3
7	1777	16	50.8	16.22	1.4	13.46	-15.9	13.18	-17.6	10.53	-34.2	15.27	-4.6	19.23	20.2	9.34	-41.6	12.52	-21.8	14.8	-7.5
8	6582	41.5	49.2	51.11	23.2	40.84	-1.4	39.79	-4.1	32.66	-21.1	46.17	11.5	60.75	46.7	28.48	-31.4	37.57	-9.5	45.84	10.7
8	10201	68	47	82.93	22	66.3	-3	64.55	-5	53	-22	74.92	10.2	98.56	45	46.21	-32	60.95	-10.4	74.38	9.4

Table 6. Experiment recorder and calculated solutions on different mathematic models (continued)

Container No.	Mass of ice (g)	Melting time (hours)	Ambient temperature (℃)	Model No. 1		Model No. 2		Model No. 3		Model No. 4		Model No. 5		Model No. 6		Model No. 7		Model No. 8		Model No. 9	
				Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)	Time hours	Error (%)
9	2377	22	49.5	21.86	0.6	17.94	-18.4	17.55	-20.2	14.13	-35.8	20.34	-7.5	25.92	17.8	12.48	-43.3	16.64	-24.4	19.84	-9.8
9	2883	24.5	52.6	24.95	1.8	20.48	-16.4	20.04	-18.2	16.13	-34.1	23.22	-5.2	29.59	20.8	14.24	-41.9	18.99	-22.5	22.65	-7.6
10	2645	25	52	34.97	40	26.35	5.4	25.44	1.7	20.51	-18	28.6	14.4	41.02	64	18.26	27	23.76	-5	30.82	23.3
11	1972	25	52	30.27	21.1	22.8	-8.8	22.01	-12	17.71	-29.2	24.72	-1.1	35.48	41.9	15.79	-36.8	20.55	-17.8	26.68	6.7
11	2004	27.3	50	32	17.2	24.11	11.7	23.27	-14.8	18.73	-31.4	26.14	-4.3	37.51	37.4	16.7	-38.8	21.73	-20.4	28.21	3.3

Model No. 1: Plane wall model with an effective area which is the inside surface area of container: $A = A_i$

Model No. 2: Plane wall model with an effective area which is the geometry average of the inside and outside area of container: $A = \sqrt{A_i \cdot A_o}$

Model No. 3: Calculate the rate of heat conduction by shape factors

Model No. 4: Spherical shell model which both inside and outside volumes of spherical shell equal to inside and outside volumes of the rectangular container

Model No. 5: Spherical shell model which inside volume of spherical shell equals to the inside volume of the container and thickness of spherical shell is the thickness of container

Model No. 6: Spherical shell model which outside volume of spherical shell equals to the outside volume of the container and thickness of spherical shell is the thickness of container

Model No. 7: Spherical shell model which both inside and outside surface areas of spherical shell equal to the inside and outside surfaces of rectangular container

Model No. 8: Spherical shell model which inside surface area of spherical shell equals to the inside surface area of the container and thickness of spherical shell is the thickness of container

Model No. 9 Spherical shell model which outside area of spherical shell equals to the outside area of the container and thickness of spherical shell is the thickness of container

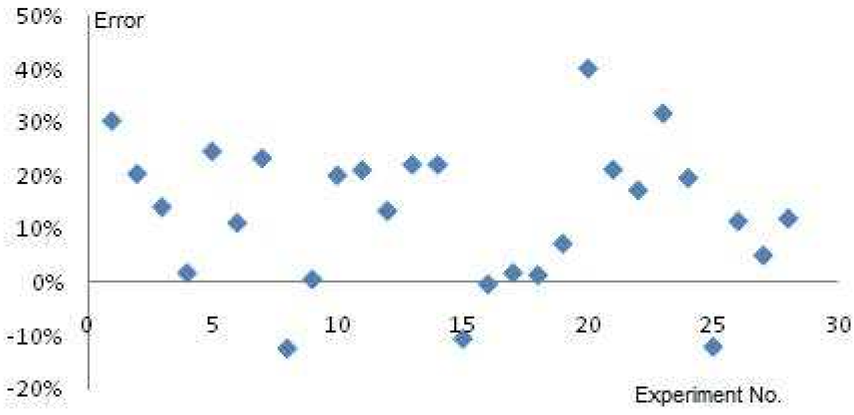


Figure 12. Scatter graph of error on model No. 1

(Average error = 13%; standard deviation = 0.1319)

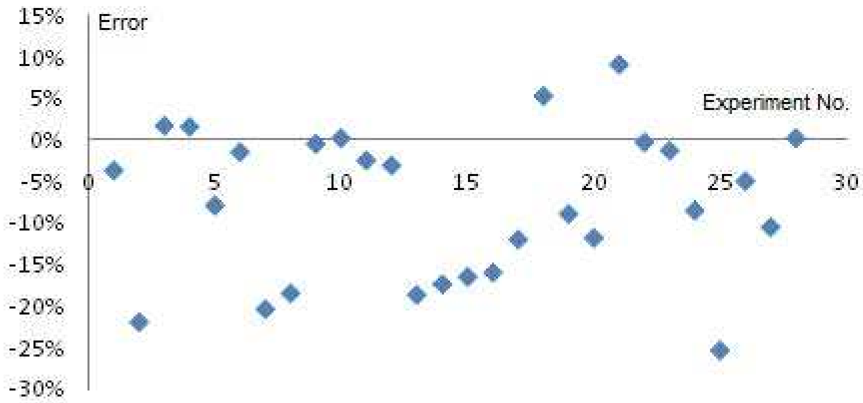


Figure 13. Scatter graph of error on model No. 2

(Average error = -8%; standard deviation = 0.0909)

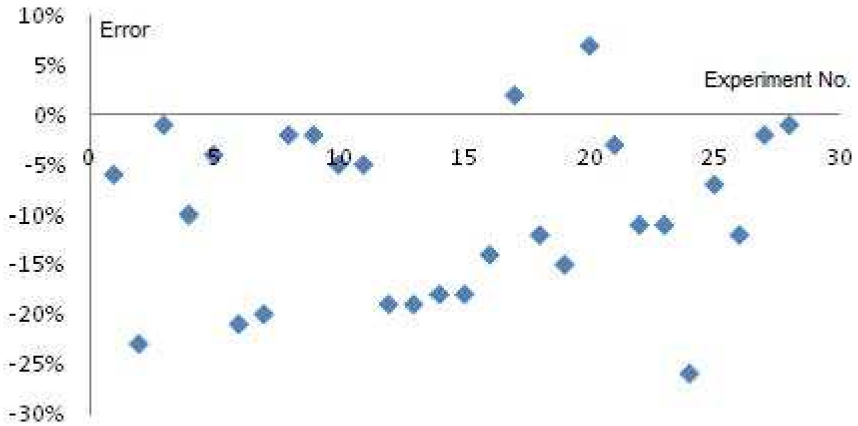


Figure 14. Scatter graph of error on model No. 3

(Average error = -10%; standard deviation = 0.0846)

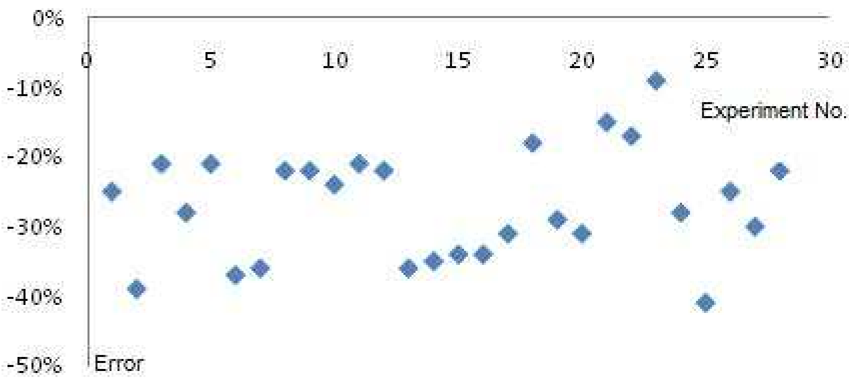


Figure 15. Scatter graph of error on model No. 4

(Average error = -27%; standard deviation = 0.0790)

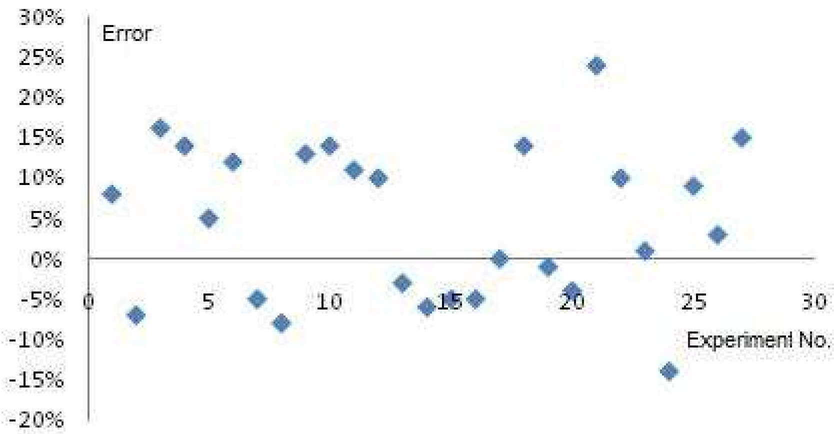


Figure 16. Scatter graph of error on model No. 5

(Average error = 4%; standard deviation = 0.0954)

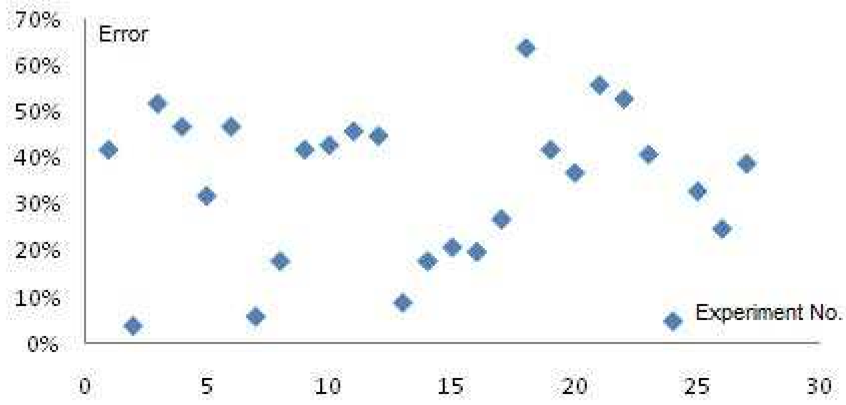


Figure 17. Scatter graph of error on model No. 6

(Average error = 34%; standard deviation = 0.1663)

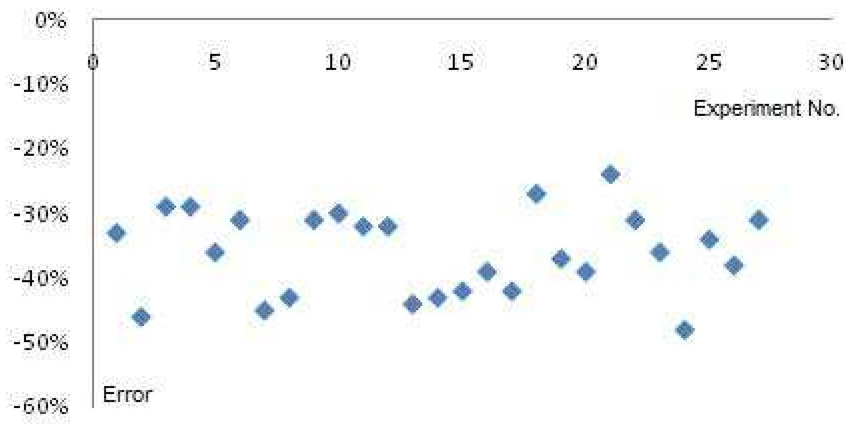


Figure 18. Scatter graph of error on model No. 7

(Average error = -36%; standard deviation = 0.0646)

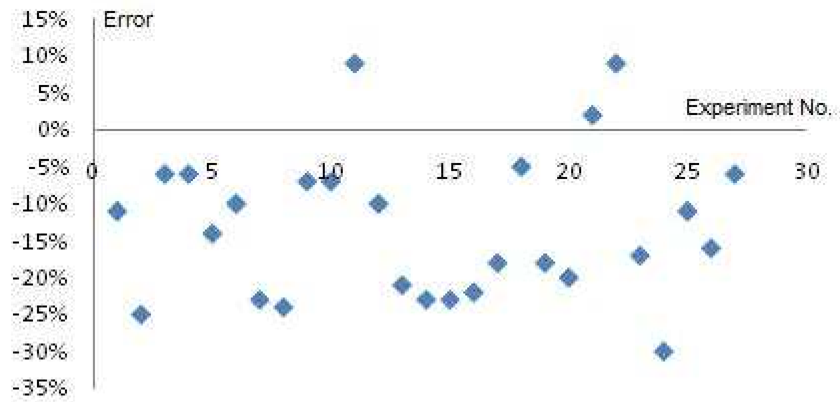


Figure 19. Scatter graph of error on model No. 8

(Average error = -13%; standard deviation = 0.1005)

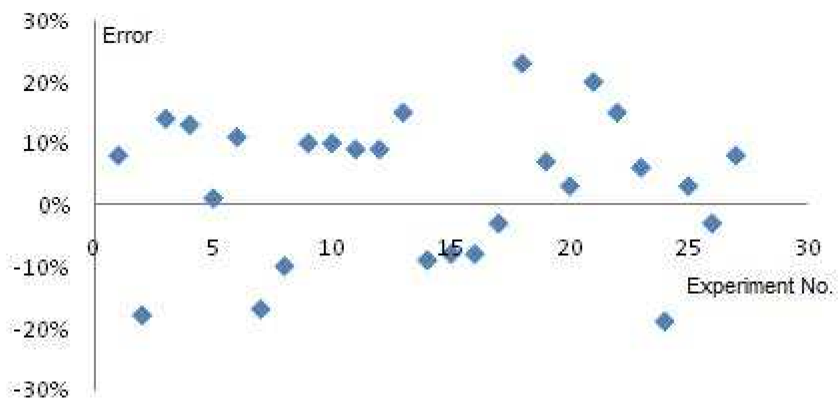


Figure 20. Scatter graph of error on model No. 9

(Average error = 3%; standard deviation = 0.1154)

Basing on Table 6, the scatter graph of errors on each model is presented as up showing as Figure 12 to Figure 20.

To evaluate the models, the absolute values of average error on each model are shown in Figure 21.

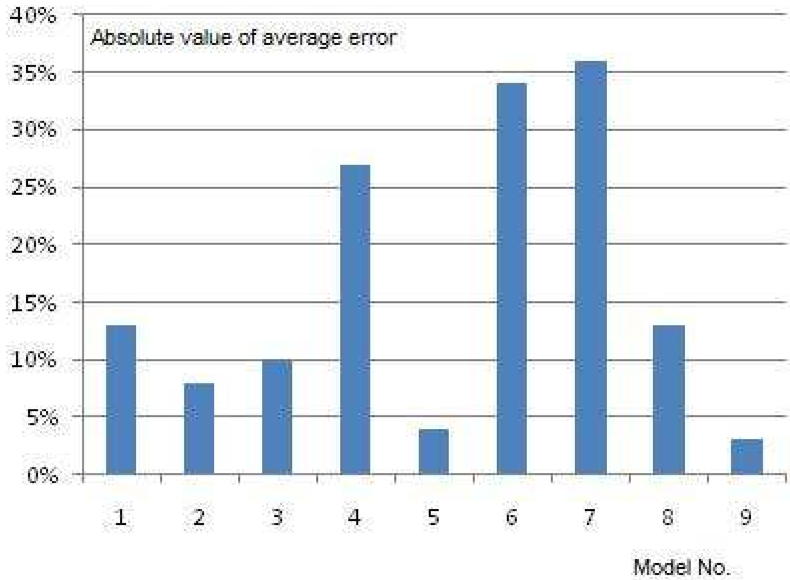


Figure 21. Absolute values of average error

It has been found that model No. 5 and No. 9 are superior to others in terms of average error. Reviewing the standard deviation of errors shown in Table 7, all values are not in significant difference. It can be concluded that the experimental data are stable and acceptable.

The average error in model No. 1 is 13%, which means this model under-predicts the duration of insulating package as longer than experimental data. In other words, model No. 1 is always conservative. It is the model that is used in most literatures.

Table 7. Standard deviations of errors

Model No.	Standard Deviation
1	0.132
2	0.091
3	0.085
4	0.079
5	0.095
6	0.166
7	0.065
8	0.101
9	0.115

The average errors in model No. 2 and No. 3 are negative values, which actually mean the duration predicted by these models is over-predicted. If the packaging designers use these models, the product may be over heated. Thus neither of these two models is acceptable.

When models No. 4 and No. 7 are used to calculate the inside and outside radius of spherical shell, the thickness of the shell (wall thickness) is thinner than the thickness of the test container. The average errors show that the duration drops quickly while the thickness decreases. From these two models it is confirmed that the thickness of the container is one of the most important parameters for insulating package design.

Based on the experimental results and the subsequent data analysis, it can be concluded that spherical shell models No. 5 and No. 9 are better than the others. Comparing these two models, the standard deviation of model No. 5 is smaller than

No. 9.

Therefore, model No. 5 will be used in the research presented hereafter. Model No. 5 is a spherical shell with the inside volume of the spherical shell equaling to the inside volume of the rectangular container and the thickness of the spherical shell equaling to the thickness of the rectangular container.

CHAPTER 4. SPHERICAL SHELL MODELS OF PACKAGING SYSTEMS

The motivation to develop a spherical shell model is to have an effective tool for packaging design, since the spherical shell model offers a closed-form mathematical solution of the heat transfer through the package. In this chapter several different spherical shell models with various levels of accuracy will be studied in application to different packaging constructions and transportation environments for insulating package design.

4.1 Mathematical model only with conduction

When a mathematical model only considers conduction, the heat transfer rate through a spherical shell can be expressed as:

$$\frac{4\pi \cdot r_o \cdot r_i \cdot k \cdot (T_o - T_i)}{r_o - r_i} = m_{ice} \cdot \Delta H / t$$
$$= \frac{4}{3}\pi \cdot \rho \cdot (r_i^{\,3} - r_p^{\,3}) \cdot \Delta H / t$$

where $r_o = r_i + \Delta x$, r_i is the inside radius of spherical shell in m, r_o is the outside radius of spherical shell in m, Δx is the thickness of the shell, k is the conductivity of the insulated material, T_i and T_o are the inside and outside surface temperatures in ℃, respectively. t is the duration of the package, r_p is the radius of the product which is converted from the product volume V:

$$r_p = (3V / 4\pi)^{1/3} \tag{4-1}$$

By manipulating the above equation, the relationship between Δx and r_i can be achieved as:

$$\Delta x = \frac{3 \cdot r_i^{\,2} \cdot k \cdot (T_o - T_i) \cdot t}{\rho \cdot (r_i^{\,3} - r_p^{\,3}) \cdot \Delta H - 3 \cdot r_i \cdot (T_o - T_i) \cdot k \cdot t} \tag{4-2}$$

To get the $r_i - \Delta x$ curve, some parameters in equation 4-2 should be determined. As a benchmark case, it is supposed that the product dimension is 125*125*90 mm^3; the distribution temperature for the product is 0 ℃; the insulating package stays at 50 ℃ ambient temperature for 48 hours (allowable shipping time); insulated material in the package is EPS.

To keep a constant distribution temperature for the product at 0 ℃, the best refrigerant is a phase change material which has a melting temperature of 0 ℃. Ice is this kind of PCM. The relative properties of ice in equation 4-2 are as follows:

$\rho = 900 \ Kg / m^3$; $\Delta H = 335000 \ J / Kg$

The radius of the product of a spherical shell model can be determined from equation 4-1:

$$r_p = (3V / 4\pi)^{1/3} = (3 \times 0.125 \times 0.125 \times 0.9 / 4\pi)^{1/3} = 0.07m$$

The conductivity of EPS is: $k = 0.042 \, w / m^2 \cdot {}^\circ c$.

Therefore, the $r_i - \Delta x$ curve can be practiced. The results are shown in

Figure 22.

Figure 22 shows the inside radius of spherical shell decrease when the thickness of wall increase. The fastest changing of the inside radius of the spherical shell happens while the thickness of shell is in a range lower than 20 mm. When the thickness of shell is thicker than 20 mm the inside radius of spherical shell changes slowly.

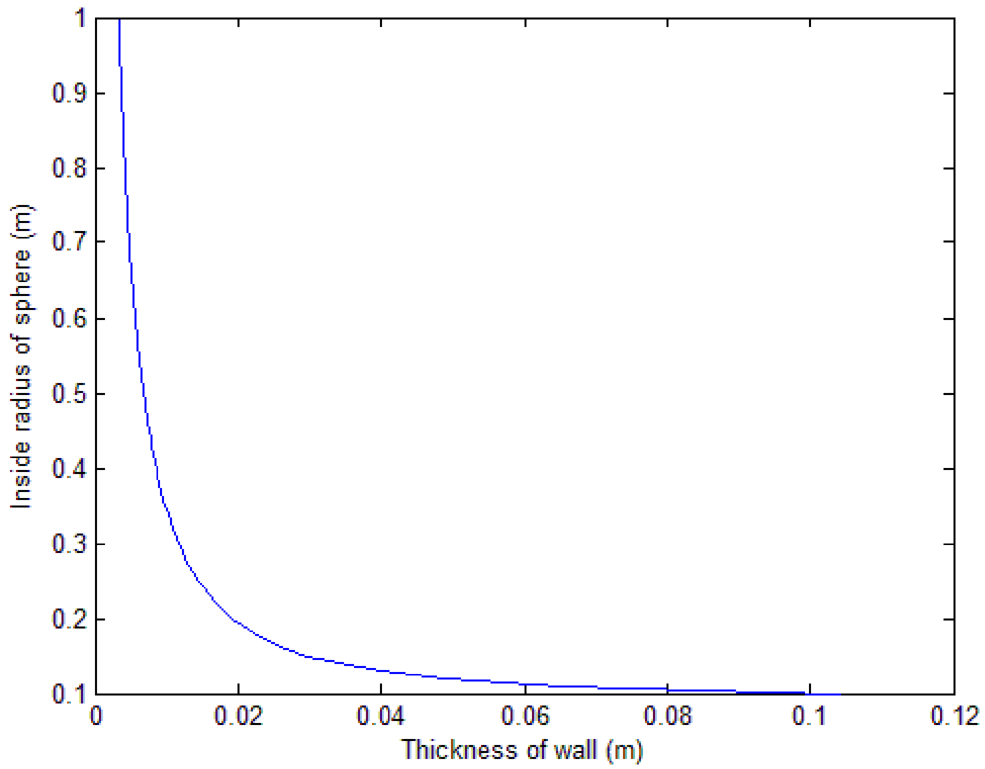


Figure 22. Inside radius - thickness curve only considering conduction

4.2 Mathematical model with external convection

Convection is classified as either natural, (free), or forced convection, depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of cooler fluid (Cengel, 2007). In most cases, involving the distribution of insulating packages, the packages are located on the floor or on the pallets which are in the warehouse or the distributing vehicle without any mechanical air-circulating device. For this reason, most of the convection heat transfers to and from insulating packages occur by natural convection.

In this study, only natural convection was considered in representing convection heat transfer. Since the insulating package is placed on the floor of the warehouse or the distributing vehicle, the outside of the package contacts a large volume of the air in the environment. Convection in this case is called “natural convection in an unconfined space”. This is the case discussed in following section.

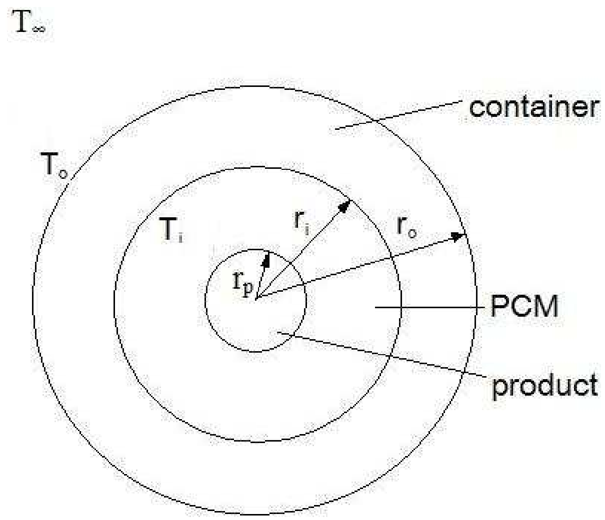


Figure 23. Sketch of spherical shell model considering convection

Figure 23 gives a sketch of the spherical shell model for an insulating package considering natural convection in an unconfined space. The outside temperature of the insulating package is T_o and the ambient temperature is T_∞. T_o and T_∞ are different.

With the spherical shell model as shown in Figure 23, a heat transfer balance can be established to yield the following equation

$$Q = h_a \cdot A_o \cdot (T_\infty - T_o) = \frac{4\pi r_o r_i k(T_o - T_i)}{r_o - r_i} \tag{4-3}$$

where h_a is the average convection heat transfer coefficient on the surface in $w/m^2 \cdot °c$; $A_o = 4\pi r_o^2$ is the outside surface area of the spherical shell in m^2; k is the conductivity coefficient of insulated material in $w/m \cdot °c$.

For an isothermal condition, the average convection heat transfer coefficient h_a for flow over a sphere can be calculated by the following equation (Yunus A. Cengel, 2007):

$$h_a = \frac{Nu \cdot k_{air}}{D_o} \tag{4-4}$$

where k_{air} is the thermal conductivity of air in $w/m \cdot °c$; D_o is the diameter of outside sphere in m; Nu is the dimensionless average Nusselt number. The Nusselt number is a function of two dimensionless numbers Ra and P_r, which are the Raleigh number and Prandtl number, respectively. For the geometry of a sphere, the average Nusselt number can be expressed by the following equation:

$$Nu = 2 + \frac{0.589 Ra^{1/4}}{[1+(0.469/P_r)^{9/16}]^{4/9}} \quad \text{when} \quad \begin{matrix} Ra \le 10^{11} \\ (P_r \ge 0.7) \end{matrix} \tag{4-5}$$

The Rayleigh number is defined by following two equations:

$$Ra = \frac{g\beta(T_o - T_\infty)D_o{}^3}{\gamma^2} P_r \tag{4-6}$$

where g is the acceleration of gravity, which is 9.82 m/s^2; γ is the viscosity of air in m^2/s; β is the volumetric coefficient of expansion of the air ($1/T_f$) in $1/K$. All air properties are evaluated at the air film temperature $T_f = (T_o + T_\infty)/2$. The conductivity of air, k, the Prandtl number, P_r, and the viscosity of air, γ, are relative with the air of film temperature and can be found in the table of properties of air.

According to the equation 4-3, the temperature of outside surface T_o can be formulated as

$$T_o = \frac{r_i k T_i + h_a \cdot \Delta x \cdot (r_i + \Delta x) \cdot T_\infty}{r_i k + h_a \cdot \Delta x \cdot (r_i + \Delta x)} \tag{4-7}$$

where Δx is the thickness of the insulated wall which is $r_o - r_i$ in m.

Assuming that all the heat transfer flow through the insulated wall is absorbed by the phase change material, the following equation can be obtained:

$$\begin{aligned} Q &= m_{ice} \cdot \Delta H / t \\ &= \frac{4}{3}\pi\rho \cdot (r_i^{\ 3} - r_p^{\ 3}) \cdot \Delta H / t \end{aligned} \tag{4-8}$$

where ρ is the density of the PCM in kg/m^3; ΔH is the latent heat of PCM in KJ / Kg ; t is the melting time of the PCM (i.e. the total lasting time of the insulating package at the specific temperature) in s; r_p is the product radius.

Therefore, from equations 4-3 and 4-8, the following formula is developed

$$Q = h_a \cdot A_o \cdot (T_\infty - T_o) = \frac{4}{3}\pi\rho (r_i^{\ 3} - r_p^{\ 3}) \cdot \Delta H / t$$

By applying equation 4-7 to the above equation, the final working equation is obtained:

$$3 \cdot t \cdot h_a \cdot k \cdot r_i \cdot (r_i + \Delta x)^2 \cdot (T_\infty - T_i) - \rho \cdot \Delta H \cdot (r_i^{\ 3} - r_p^{\ 3}) \cdot [r_i k + h_a \cdot \Delta x \cdot (r_i + \Delta x)] = 0 \tag{4-9}$$

This equation shows the relationship between the inside radius r_i and the thickness of the spherical shell (the thickness of insulated container) Δx when the geometric dimension of product r_p and the shipping time t are given, and the phase change material and insulation material are chosen. The inside radius r_i of the shell is related to the quantity of phase change material which is needed, and, working together with thickness Δx the volume of the insulating package can be determined, which partially determines the shipping cost.

All parameters in equation 4-9 are the same as in section 4.1.

$t = 48 hours$; $T_\infty = 50°C$; $\rho = 900 Kg/m^3$;
$\Delta H = 335000 J/Kg$; $k = 0.042 w/m \cdot °C$; $T_i = 0°C$

Now using equations 4-4 to 4-6, h_a can be calculated as the following:

$$T_\infty = 50 + 273 = 323°K; \quad T_i = 273°K$$

Supposing the temperature of outside package surface is at $24°c(297K)$ and the possible largest outside diameter of the package r_o is $0.2m(D_o = 0.4m)$.

Therefore, the film temperature will be:

$$T_f = (T_\infty + T_o)/2 = (323 + 297)/2 = 310K$$

The properties of air at the film temperature and at 1 ATM pressure are:

$$k_{air} = 0.026998\ w/m \cdot °c \qquad P_r = 0.7058$$
$$\nu = 1.6696 \times 10^{-5} m/s^2 \qquad \beta = \frac{1}{T_f} = \frac{1}{310K}$$

The Rayleigh number becomes: (equation 4-6)

$$Ra = \frac{g\beta(T_o - T_\infty)D_o^3}{\gamma^2} P_r = \frac{(9.82m/s^2)(1/310K)(50-23K)(0.4m)^3}{(1.6696\times10^{-5}m^2/s)^2}(0.7058)$$
$$= 1.385\times10^8$$

The natural convection Nusselt number at this condition can be determined from equation 4-5.

$$Nu = 2 + \frac{0.589Ra^{1/4}}{[1+(0.469/P_r)^{9/16}]^{4/9}} = 2 + \frac{0.589\times(1.385\times10^8)^{1/4}}{[1+(0.469/0.7058)^{9/16}]^{4/9}}$$
$$= 51.27$$

Then, by using equation 4-4, h_a:

$$h_a = \frac{Nu \cdot k_{air}}{D_o} = \frac{51.27\times0.026998}{0.4} = 3.46 w/m^2 \cdot °c$$

Applying all these calculated parameters to equation 4-9, the relationship between the inside radius r_i and the thickness of spherical shell Δx is achieved (as shown in Figure 24).

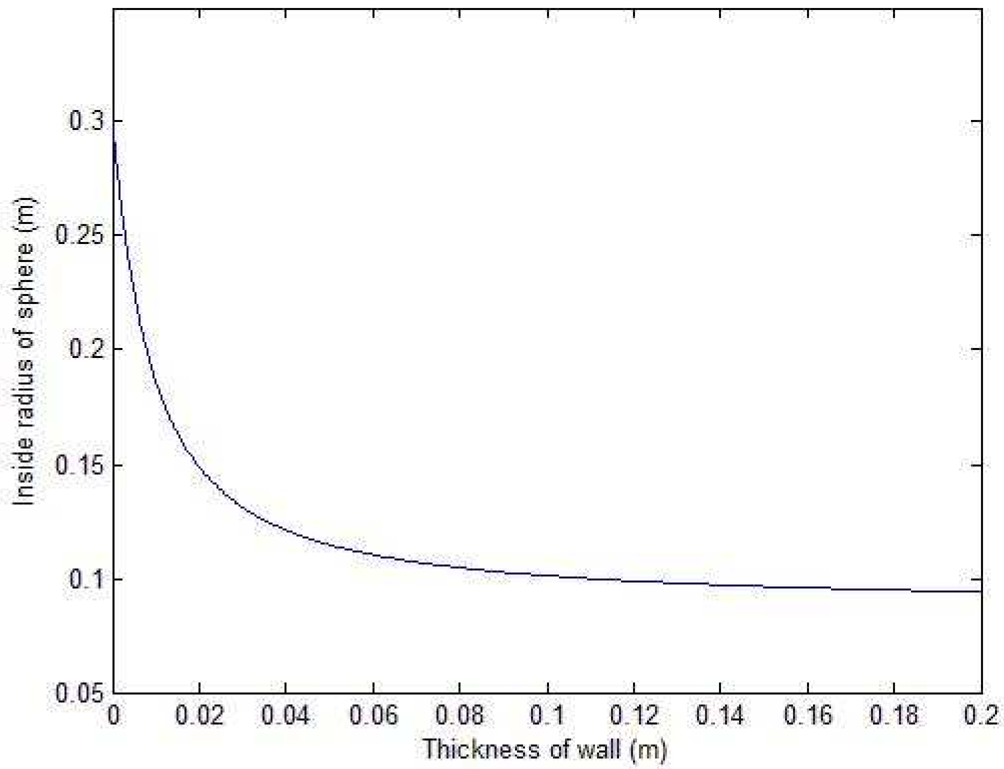


Figure 24. Thickness of wall –inside radius of shell curve considering convection

In Figure 24, the $r_i - \Delta x$ curve shows the inside radius of spherical shell decrease when the thickness of wall increase. The steepest decrease of the inside radius of spherical shell happens while the thickness of shell is in the range lower than 0.06 m. When the thickness of shell is higher than 0.06m the inside radius of spherical shell changes slowly. The inside radius of the shell can directly be used to determine the quantity of the PCM in the insulating package ($m = \frac{4}{3}\pi\rho(r_i^3 - r_p^3)$). In other words, the dosage of the PCM in insulating package can be significantly decreased when the wall of the insulating container gets thicker. However once the thickness of the wall reaches a critical value, further increase in the thickness of the shell will no longer reduce the dosage of the PCM.

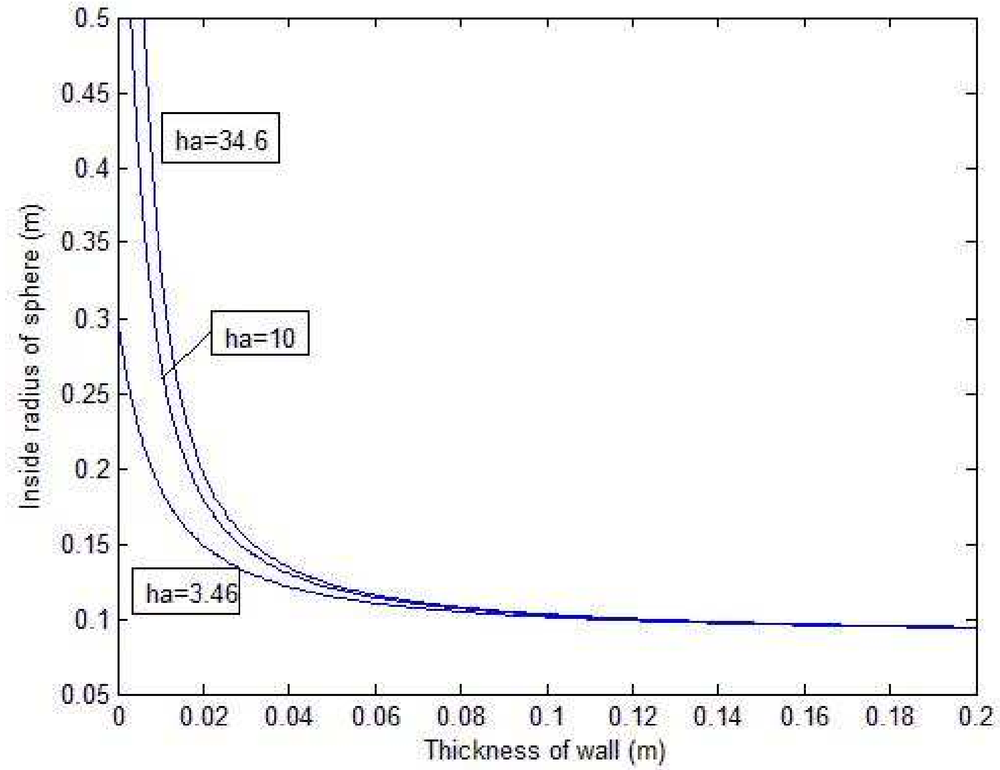


Figure 25. Thickness of wall –inside radius of shell curve with h_a change

Figure 25 shows that natural convection in an unconfined space is pronounced when the thickness of wall is thin. This means the basic thickness of container is essential when the insulating package is first designed. The minimum thickness of insulating package should be more than 30 mm in order to prevent convection from significantly affecting the total heat gain during shipment. The conclusion of this thickness requirement found from this research is very much consistent with what is presently practiced in packaging industry. As a result, the effect of outside convection on the insulating package can be limited and the quantity of PCM will not be changed significantly when convection is neglected, provided the wall is thicker than a critical value.

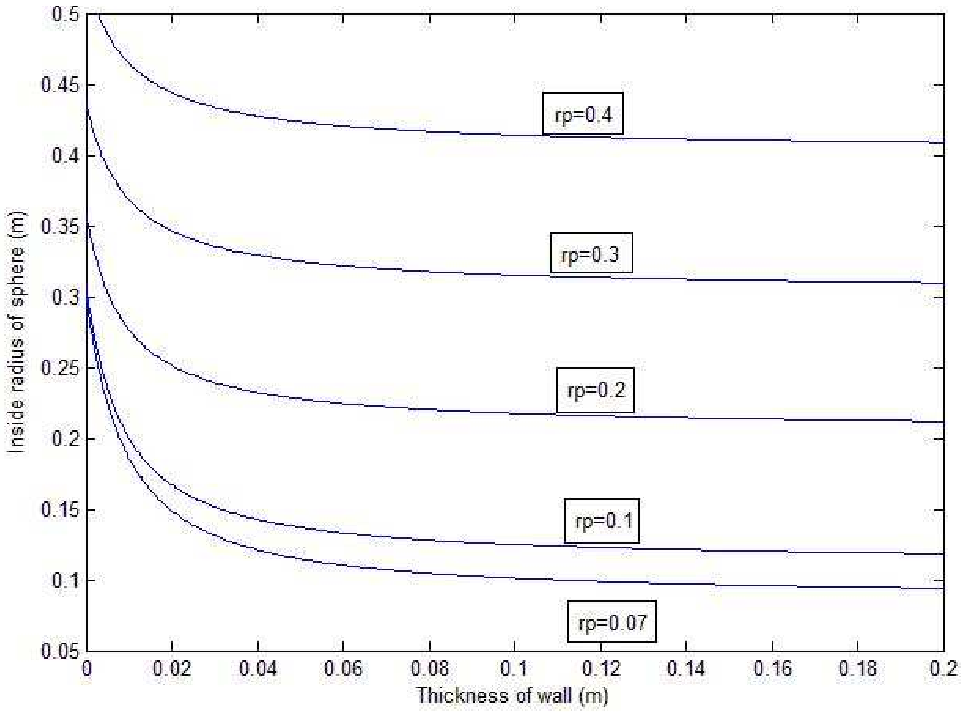


Figure 26. Thickness of wall –inside radius of shell curve with r_p change

Figure 26 reveals the fact that the inside radius of the shell is proportional to the radius of product. Bigger products need greater quantities of refrigerate to keep the shipping temperature in a proper range for a given duration.

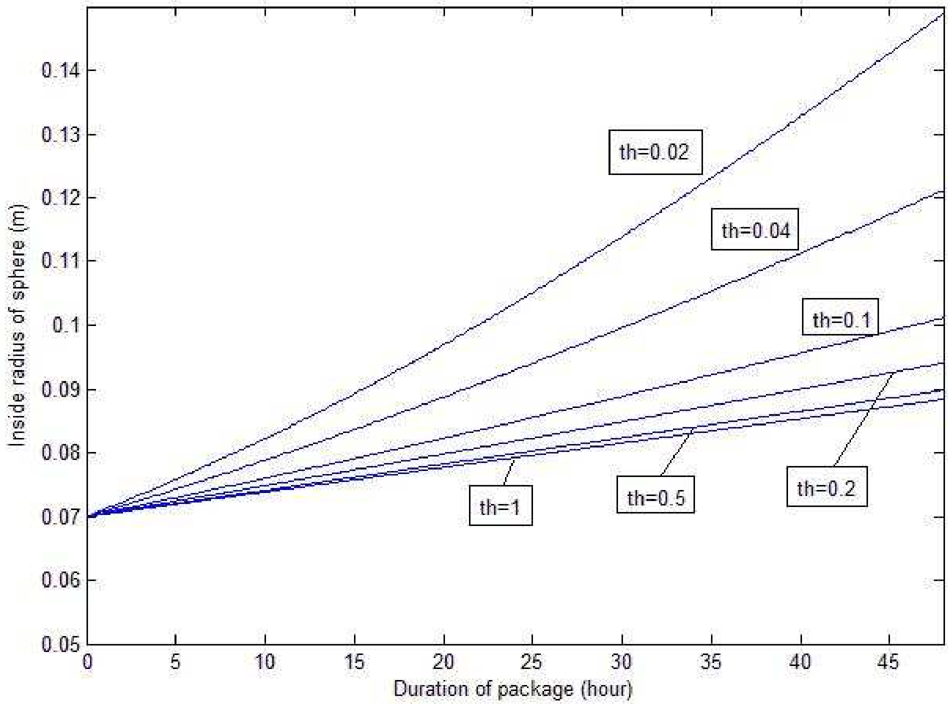


Figure 27. Inside radius-duration of package curve with thickness of wall change

Figure 27 shows that longer durations of packages need a bigger inside radius of the shell, subsequently with more PCM for a given size product. The changes of these quantities are approximately linear. Thicker thickness of the wall can significantly reduce the quantity of PCM before a critical point. Any further increasing of thickness is less effective to cut the quantity of PCM down when the thickness of the wall is over 40 mm.

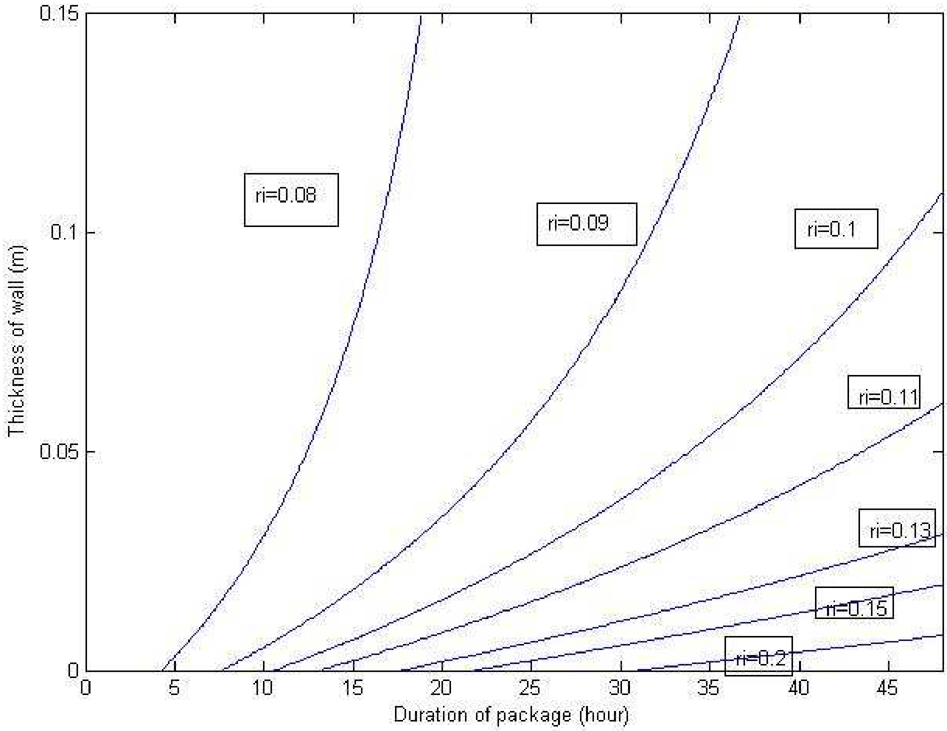


Figure 28. Thickness of wall-duration of package curve with r_i change

Figure 28 shows that thicker walls can extend the duration of package when the quantity of PCM is constant. To achieve a longer duration the most effective methods are increasing the thickness of the wall and increasing the quantity of PCM.

4.3 Mathematical model with external convection and radiation

When an insulating package is transported on a highway in summer, the inside temperature of the truck trailer can be over $60℃$. In this case, not only must convection be considered but radiation cannot be neglected. Normally, to reduce the effect of radiation, the insulating package is covered by some high emissivity material (such as aluminum foil). In this section a mathematical model will be developed to simulate the distribution environment with both convection and radiation.

Air is nearly transparent to radiation, and thus heat transfer through air to the package is by simultaneous convection and radiation. Natural convection heat transfer coefficients are typically very low, compared to forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it should be considered in natural convection problems like the one at hand. The total rate of heat transfer is determined by adding the convection and radiation components:

$$Q_{total} = Q_{conv} + Q_{rad}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_∞ (both in K) is determined from

$$Q_{rad} = \varepsilon\sigma A_s (T_\infty{}^4 - T_s{}^4) \tag{4-10}$$

where T_s is the temperature of the surface in K, ε is the dimensionless surface's emissivity, $\sigma = 5.670 \times 10^{-8} W/m^2 \cdot K^4$ is the Stefan-Boltzmann constant, and A_s is the surface area.

With the combination of convection and radiation on the spherical shell model, the heat transfer from outside to inside of insulating package can be expressed by the following equation:

$$h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon\sigma A_o (T_\infty{}^4 - T_o{}^4) = \frac{4\pi r_o r_i k(T_o - T_i)}{r_o - r_i}$$

$$= \frac{4}{3}\pi\rho(r_i{}^3 - r_p{}^3) \cdot \Delta H / t \tag{4-11}$$

From equation 4-11, the temperature of the surface on a spherical shell model T_o can be expressed as follows.

$$T_o = T_i + \frac{\rho \cdot (r_i{}^3 - r_p{}^3) \cdot \Delta H \cdot \Delta x}{3t \cdot (r_i + \Delta x) \cdot r_i \cdot k} \tag{4-12}$$

Substituting equation 4-12 into equation 4-11, the following relation can be achieved:

$$3t \cdot h_a \cdot (r_i + \Delta x)^2 (T_\infty - T_i - \frac{\rho \cdot (r_i^3 - r_p^3) \cdot \Delta H \cdot \Delta x}{3t \cdot (r_i + \Delta x) \cdot r_i \cdot k})$$
$$+ 3t \cdot \varepsilon \cdot \sigma \cdot (r_i + \Delta x)^2 [T_\infty^4 - (T_i + \frac{\rho \cdot (r_i^3 - r_p^3) \cdot \Delta H \cdot \Delta x}{3t \cdot (r_i + \Delta x) \cdot r_i \cdot k})^4] \qquad (4\text{-}13)$$
$$= \rho \cdot (r_i^3 - r_p^3) \cdot \Delta H$$

Equation 4-13 shows the relationship between the inside radius r_i and the thickness of the shell Δx when both the outside convection and radiation are considered.

Normally, the emissivity of EPS foam is 0.9. The other parameters in equation 4-13 are taken as following:

$t = 48hours$; $h_a = 3.46 w/m^2 \cdot °c$; $T_\infty = 50°C$; $\rho = 900 Kg/m^3$; $\Delta H = 335000 J/Kg$; $k = 0.042 w/m \cdot °C$; $T_i = 0°C$

Figure 29 shows the relationship between the inside radius of the shell and the thickness of the shell when the combined convection and radiation is considered. Before the thickness of the shell reaches 40 mm, increasing the shell thickness can significantly decrease the inside radius of the shell (the quantity of the PCM as well).

Figure 30 presents a fact that the convection coefficient is very sensitive to the quantity of PCM when the thickness of the shell is less than 5 mm. However, when the thickness of the shell is over 40 mm, the effect of convection can be neglected.

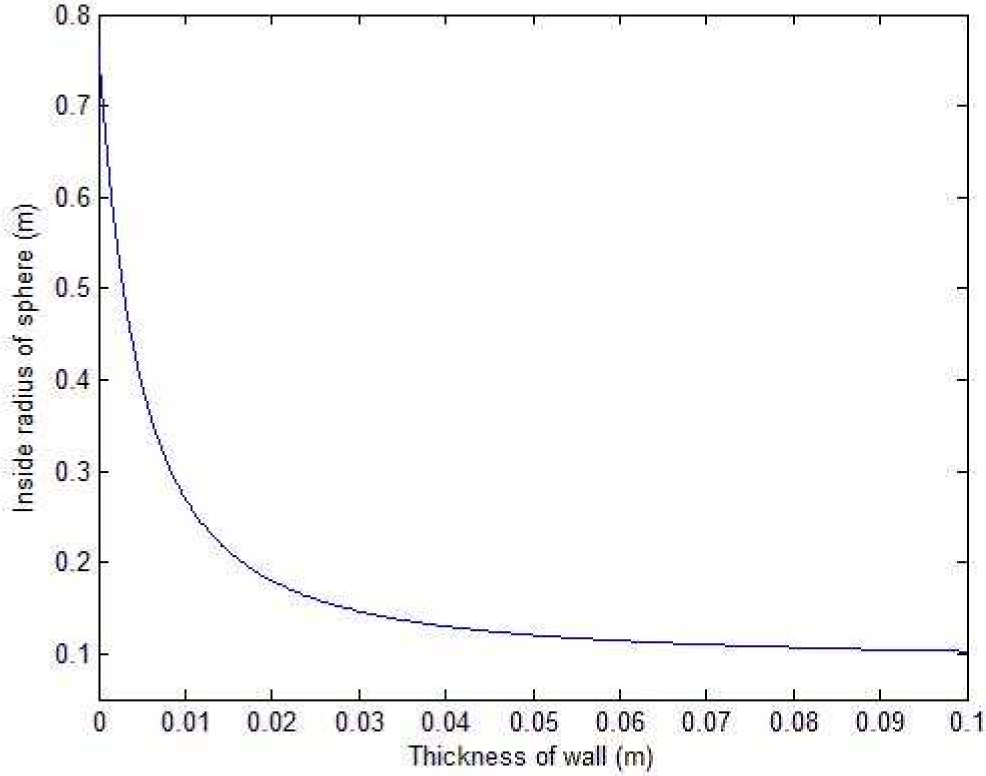


Figure 29. Thickness of wall –inside radius of shell curve considering convection and radiation

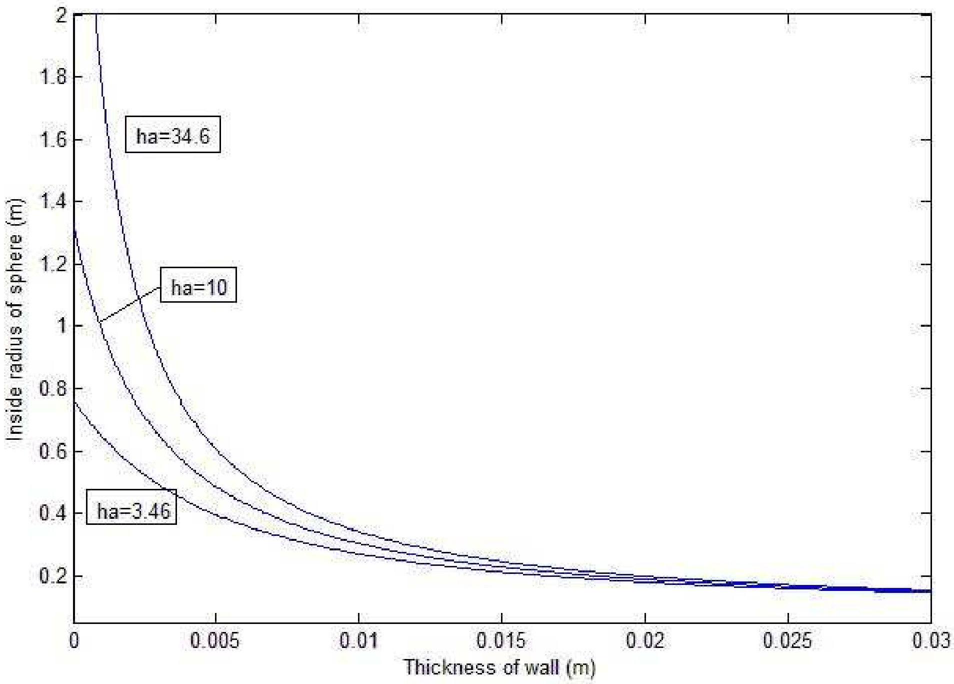


Figure 30. $r_i - \Delta x$ curves with h_a change considering convection and radiation

Figure 31 shows that usage of high emissivity material can play a significant role in reducing the quantity of PCM when the thickness of the shell is less than 40 mm. So being covered with high emissive material(such as aluminum foil) on the surface of insulating packge can decrease the effect of radiation. When $\varepsilon = 0$, this make a situation in which radiation is neglected. Figure 31, it indicates that the outside radiation cannot be ignored when the thickness of the shell is less than 40 mm.

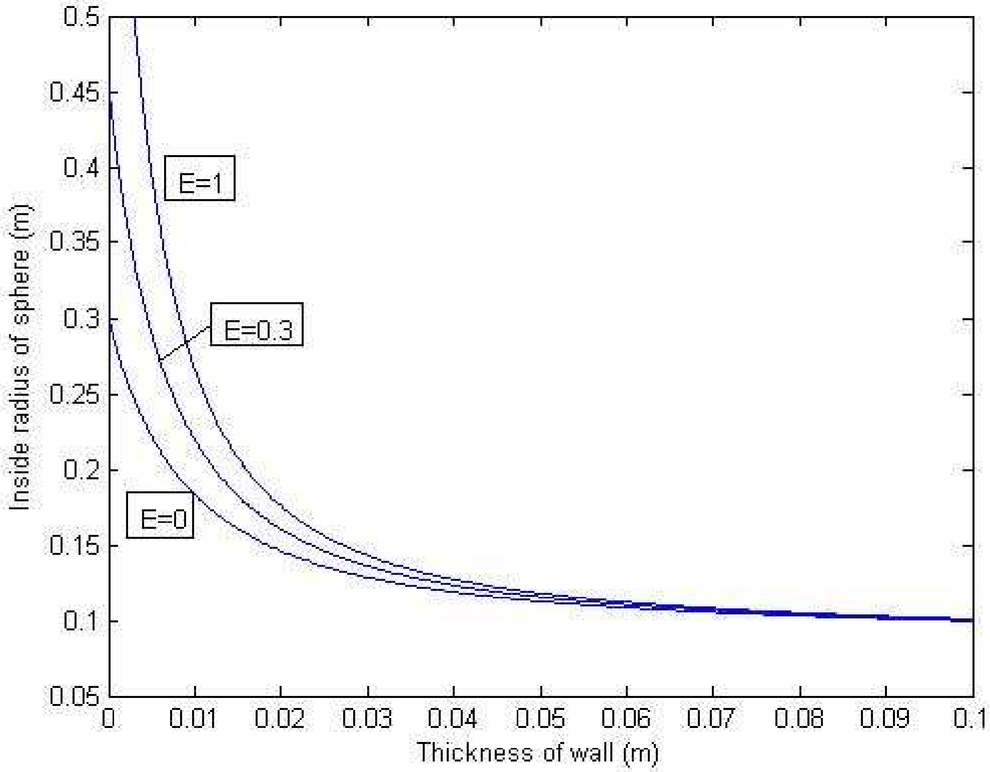


Figure 31. $r_i - \Delta x$ curves with ε change considering convection and radiation

Figure 32 presnts the $r_i - \Delta x$ curves when the size of the product changes. The result shows that the larger the product is, the greater is the amount of PCM needed.

Figure 33 gives the $r_i - \Delta x$ curves when the ambient temperature is changed. It shows that at a higher ambient temperature, the heat gain of the package needs to be offset either with a thicker container or with a greater PCM dosage.

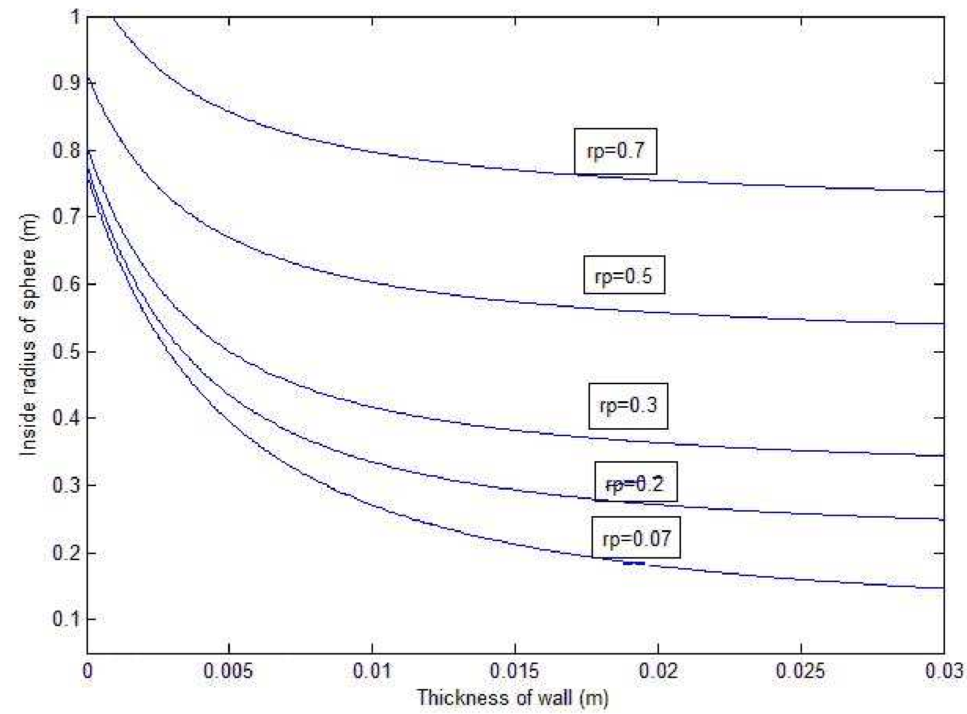


Figure 32. $r_i - \Delta x$ curves with r_p change considering convection and radiation

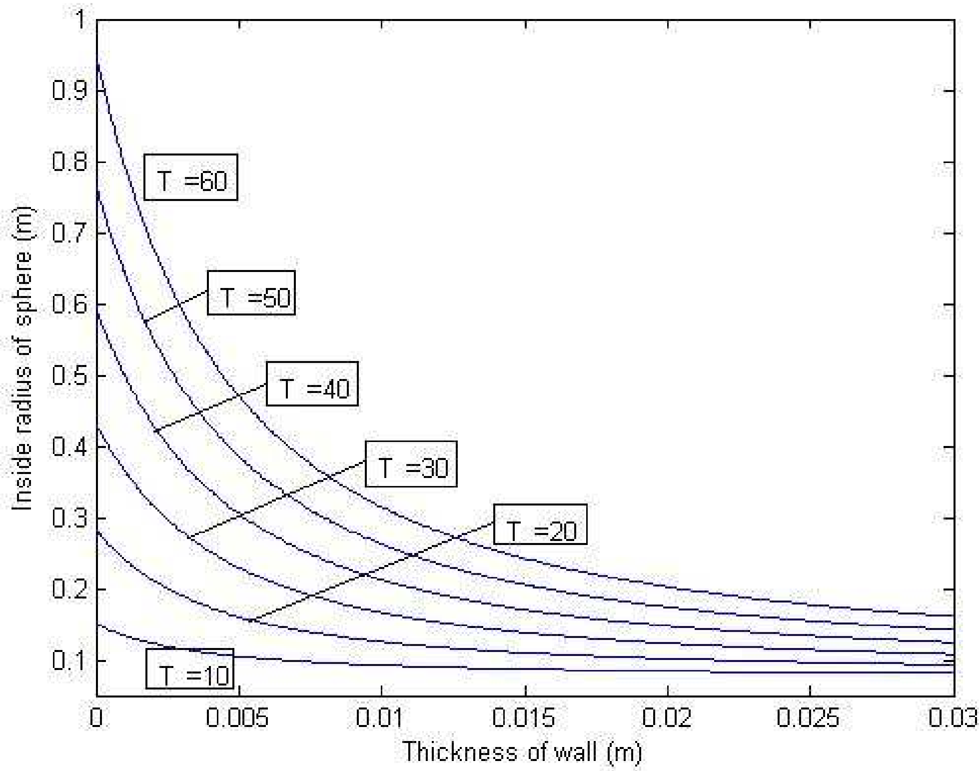


Figure 33. $r_i - \Delta x$ curves with T_∞ change considering convection and radiation

Figure 34 shows that the longer duration a package needs the larger is the needed inside radius of the shell, i.e. more PCM for a given product. The duration varies almost linearly with the inside radius of the shell. Again, a thicker shell can significantly reduce the dosage of PCM at first, while later increasing the thickness has less effect on the dosage of PCM when the thickness of the shell exceeds 40 mm.

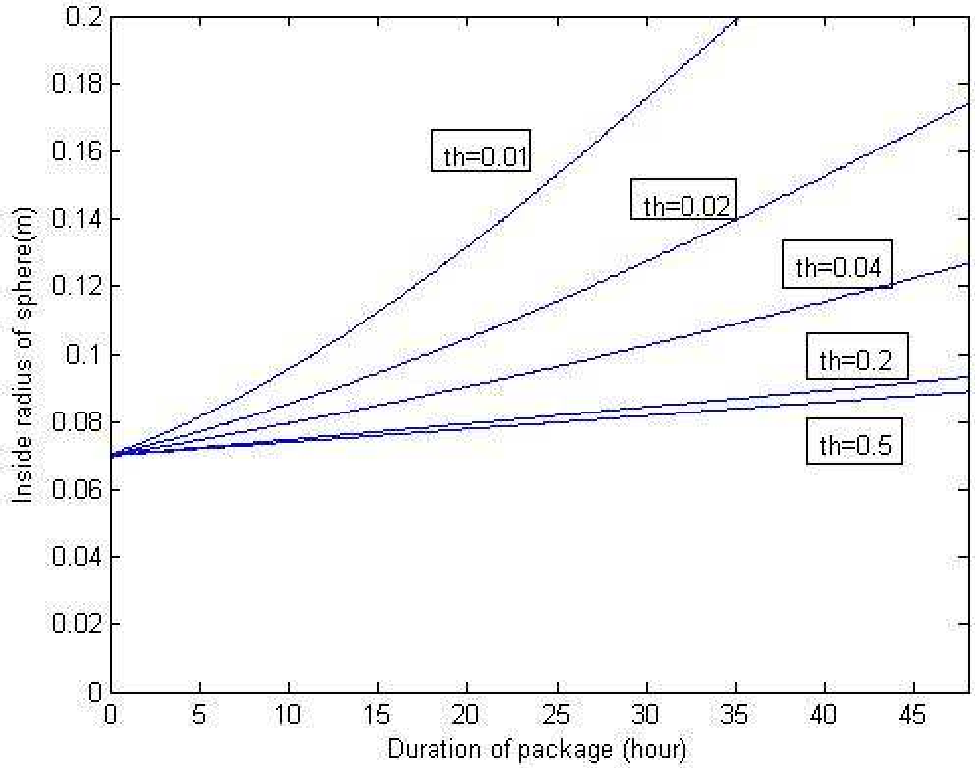


Figure 34. $r_i - t$ curves with thickness of wall change considering convection and radiation

In Figure 35, these three curves, which express the inside radius as against the thickness of the shell, represent predictions based on the model of considering only conduction, convection and combined convection and radiation, respectively. The conclusion is that when the thickness of container more than 40 mm all calculated solutions can be simplified as just considering conduct and ignoring the effect of convection and radiation.

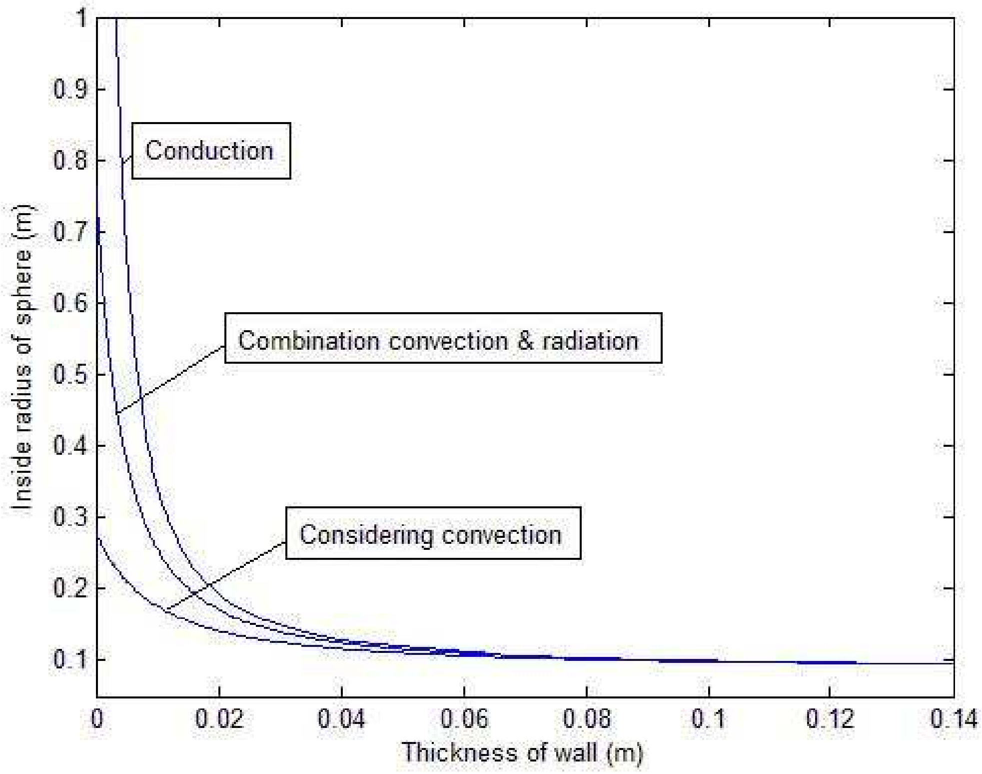


Figure 35. Inside radius - thickness curves considering different heat transfer mode

4.4 Mathematical model of multi-layered wall

In the previous section, basic theory of heat transfer and applications to insulating packages was introduced. To simplify the procedure, the terminology of thermal resistance of material is used to calculate heat transfer through a multi-layer wall of the insulating package. If the wall consists of a single material, it is very simple to calculate the thermal resistance of the package. In this case, the thermal resistance of the package is the reciprocal of the insulated material's conductivity. However, insulating packages used in many applications have much more complicated structures. Many insulating packages use a multi-layered structure with combinations of several insulated materials. In many cases, these layered materials are loosely fitted to each other to obtain extra thermal resistance by entrapping air between the layers of insulated materials. A loose-fitting EPS foam jacket inside a corrugated box is a good example of this kind of case. Since the thickness of air is very small (normally no more than 10 mm), the main heat transfer through a thin air layer is conduction (when the air gap is not more than 16-19mm, there is no convection current loss), and the enclose convection and radiation can be ignored.

When the spherical shell model is used to simulate a rectangular container and the insulating packaging has a multi-layer wall whose construction is like that shown in Figure 36. There is a layer of air between two different insulated material layers. If a wall has n layers, then, there are in total n-1 air spaces between them.

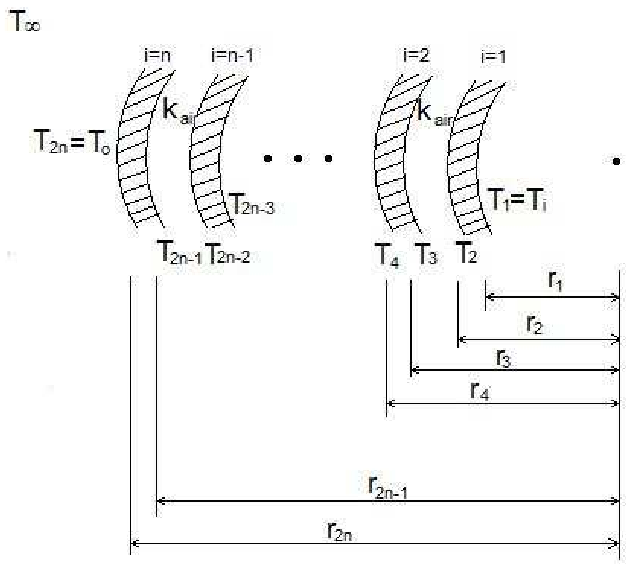


Figure 36. Multi-layered wall insulating container

Under the spherical shell model, the equation for heat flow across each layer is

$$Q = \frac{4\pi r_1 r_2 k_1 (T_1 - T_2)}{r_1 - r_2} \quad ;$$

$$Q = \frac{4\pi r_2 r_3 k_{air} (T_2 - T_3)}{r_2 - r_3};$$

…

$$Q = \frac{4\pi r_{2n-1} r_{2n} k_n (T_{2n-1} - T_o)}{r_{2n-1} - r_{2n}}$$

Therefore, the thermal resistance of each layer is

$$R_1 = \frac{r_2 - r_1}{4\pi r_1 r_2 k_1};$$

$$R_2 = \frac{r_3 - r_2}{4\pi r_2 r_3 k_{air}};$$

$$\ldots$$

$$R_{2n-1} = \frac{r_{2n} - r_{2n-1}}{4\pi r_{2n-1} r_{2n} k_n}$$

The total heat transfer through a multi-layered wall is

$$Q = \frac{T_1 - T_{2n}}{\sum R_i}$$

$$= \frac{T_i - T_o}{\frac{r_2 - r_1}{4\pi r_1 r_2 k_1} + \frac{r_4 - r_3}{4\pi r_3 r_4 k_2} + \ldots + \frac{r_{2n} - r_{2n-1}}{4\pi r_{2n-1} r_{2n} k_n} + \frac{1}{k_{air}} \left(\frac{r_3 - r_2}{4\pi r_2 r_3} + \frac{r_5 - r_4}{4\pi r_4 r_5} + \ldots + \frac{r_{2n-1} - r_{2n-2}}{4\pi r_{2n-2} r_{2n-1}} \right)}$$

(4-14)

4.5 Mathematical model considering enclosed radiation

When a high emissivity material (such as aluminum foil) is used on certain inside surfaces of multi-layered insulating wall (see Figure 37) with the spherical shell model, the net rate of radiation heat transfer between the concentric spheres can be expressed by following equation:

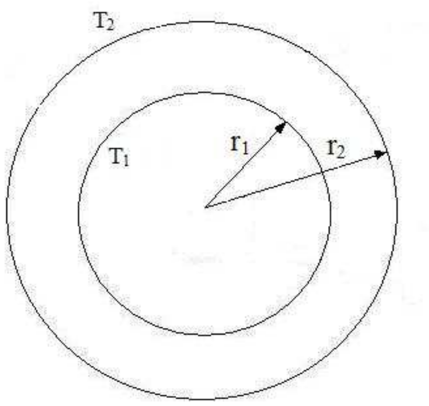


Figure 37. Concentric sphere

$$Q_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}(\frac{r_1}{r_2})^2} \tag{4-15}$$

In this case, the heat transfer from outside to inside insulating package (Figure 38) can be determined by the spherical shell model as:

$$Q = \frac{T_o - T_2}{\sum R_{2i}} = \frac{A_1\sigma(T_2^{\,4} - T_1^{\,4})}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}(\frac{r_1}{r_2})^2} + \frac{4\pi r_1 r_2 k_{air}(T_2 - T_1)}{r_2 - r_1} = \frac{T_1 - T_i}{\sum R_{1i}} \tag{4-16}$$

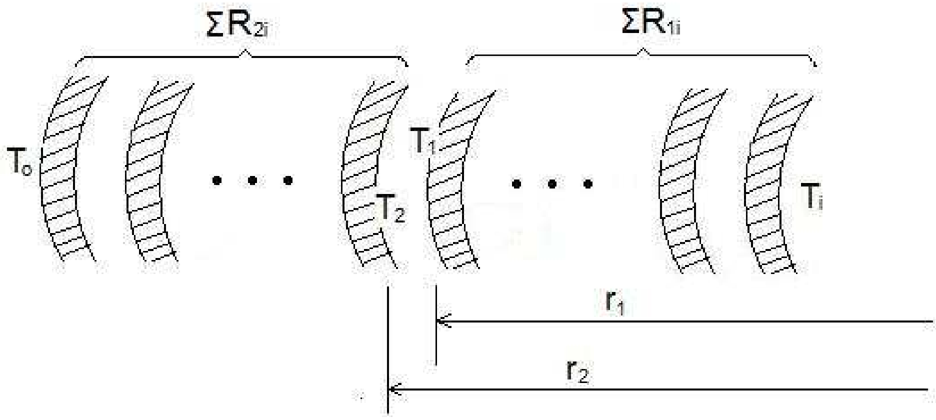


Figure 38. Mathematical model considering enclosed radiation

where $\sum R_{2i}$ is the total thermal resistance of multi-layered walls beyond the outside surface 2 of high emissivity material; $\sum R_{1i}$ is the total thermal resistance of multi-layered walls beyond the inside surface 1 of a high emissivity material.

CHAPTER 5. EXPERIMENT VALIDATION

5.1 Experiment validation on packaging models

Conduction, convection and radiation are the three modes of heat transfer. When the insulating package is transported for commercial delivery, all these modes will be involved. Three mathematical models in which these heat-transfer modes are considered partially or completely will be explored in this chapter.

The experimental verification was done just as in chapter 3. The duration of packaging is real data that can be obtained directly from the experiments. In order to compare the predictions from the mathematical models with the experimental data, the durations of package from different models are presented as follows.

5.1.1 Single-wall models

Calculations on the single-wall packaging model were mode and presented in chapter 3 and chapter 4. Here the effort is to integrate them and make further analysis.

1. Single-wall packaging model in which only conduction is considered;

$$t = \frac{m \cdot \Delta H \cdot (r_o - r_i)}{4\pi \cdot r_o \cdot r_i \cdot k \cdot \Delta T} \qquad (3\text{-}12)$$

2. Single-wall packaging model in which both conduction and convection are considered;

After reviewing the equation4-3, 4-7 and 4-8, it can be recast to get the following form of the equation for duration of the packaging:

$$t = \frac{m \cdot \Delta H \cdot [r_i k + h_a \cdot \Delta x \cdot (r_i + \Delta x)]}{4\pi \cdot h_a \cdot k \cdot r_i \cdot (r_i + \Delta x)^2 \cdot (T_\infty - T_i)} \qquad (5\text{-}1)$$

Unfortunately, the natural convection coefficient h_a is a parameter that depends on the ambient temperature T_∞, the surface temperature of insulating package T_o and the dimension of the packaging system. From experimental data, it has already known that the temperature difference between T_∞ and T_o is not more than 6℃when the ambient temperature is about 50℃(see Figure 39 and 40).

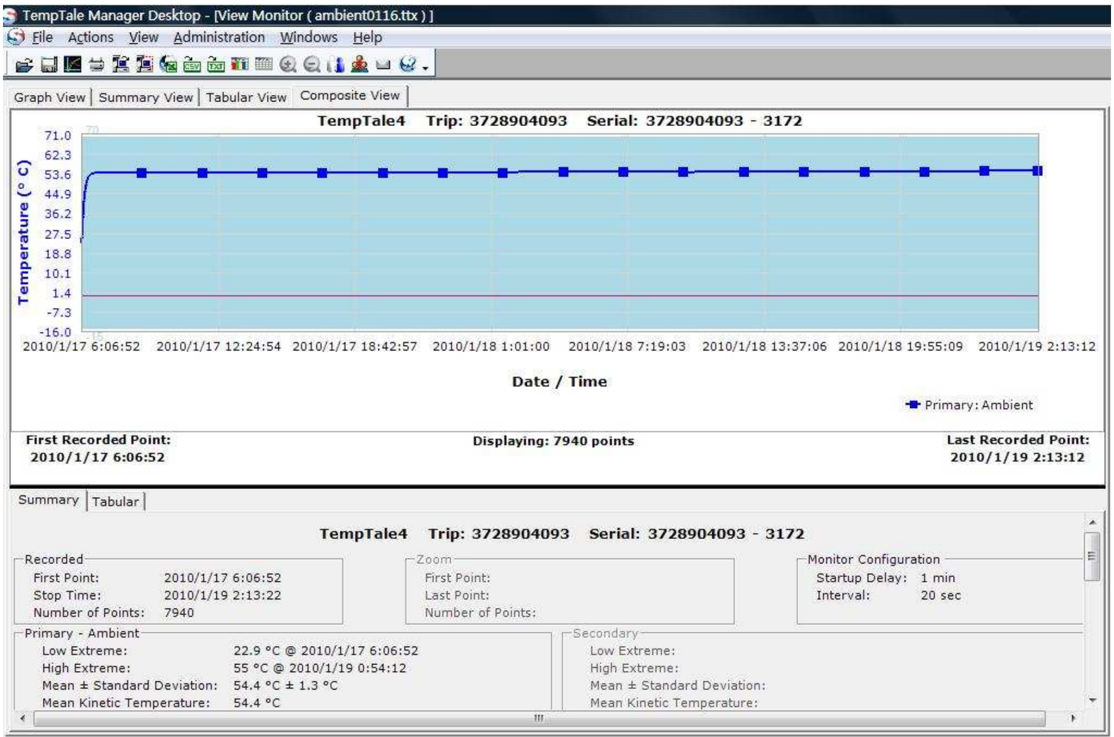


Figure 39. Ambient temperature data

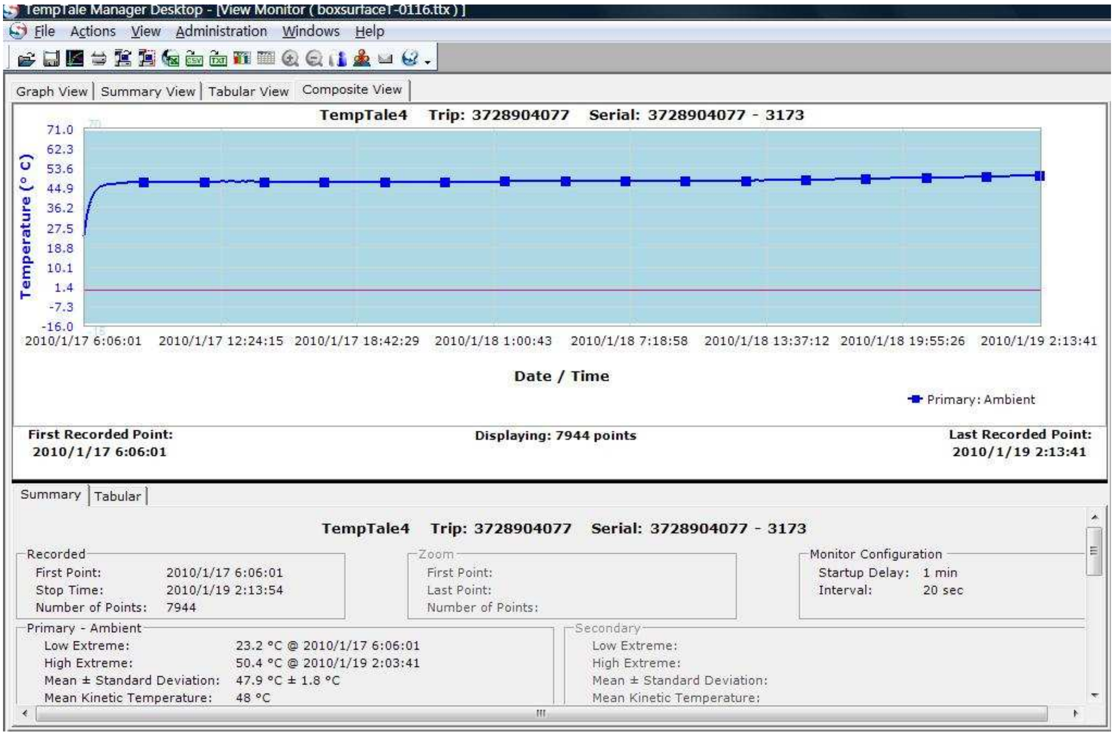


Figure 40. Temperature of container outer surface

Therefore, h_a should be recalculated as follows:

$T_\infty = 50 + 273 = 323°K$; $T_o = 44 + 173 = 317°K$; $T_i = 273°K$

Assuming the temperature of the outside package surface is 44℃ $(317°K)$ and the largest outside diameter of the package r_o is $0.2m(D_o = 0.4m)$, the film temperature will be:

$$T_f = (T_\infty + T_o)/2 = (323 + 317)/2 = 320°K$$

The properties of air at the above film temperature and at 1.0 ATM pressure are:

$$k_{air} = 0.027756 w/m\cdot°c \qquad P_r = 0.7036$$

$$\nu = 1.7712\times10^{-5} m/s^2 \qquad \beta = \frac{1}{T_f} = \frac{1}{320K}$$

The Rayleigh number becomes: (equation 4-5)

$$Ra = \frac{g\beta(T_o - T_\infty)D_o^{\ 3}}{\gamma^2}P_r = \frac{(9.82m/s^2)(1/320K)(50-44K)(0.4m)^3}{(1.7712\times10^{-5}m^2/s)^2}(0.7036)$$

$$= 2.6\times10^7$$

The natural convection Nusselt number in this case can be determined from equation 4-4 as:

$$Nu = 2 + \frac{0.589Ra^{1/4}}{[1+(0.469/P_r)^{9/16}]^{4/9}} = 2 + \frac{0.589\times(0.26\times10^8)^{1/4}}{[1+(0.469/0.7036)^{9/16}]^{4/9}}$$

$$= 34.4215$$

Then,

$$h_a = \frac{Nu \cdot k_{air}}{D_o} = \frac{34.4215 \times 0.027756}{0.4} = 2.3885 w / m^2 \cdot °c$$

3. Single-wall model, considering conduction, convection and radiation

From equation 4-11, one can derive the equation for determining duration of insulating package which is a quadratic equation. Mathematic software (Matlab) is used to get the solution from the implicit form of the equation as below,

$$\begin{aligned} & t \cdot h_a \cdot (r_i + \Delta x)^2 (T_\infty - T_i - \frac{m \cdot \Delta H \cdot \Delta x}{4\pi \cdot t \cdot (r_i + \Delta x) \cdot r_i \cdot k}) \\ & + t \cdot \varepsilon \cdot \sigma \cdot (r_i + \Delta x)^2 [T_\infty{}^4 - (T_i + \frac{m \cdot \Delta H \cdot \Delta x}{4\pi \cdot t \cdot (r_i + \Delta x) \cdot r_i \cdot k})^4] \\ & = \frac{m \cdot \Delta H}{4\pi} \end{aligned} \quad (5\text{-}2)$$

As presented in chapter 3, rectangular containers are represented to spherical shells whose inside volume equal to the inside volume of the rectangular container and thickness equal to the thickness of the rectangular container. Table 8 shows this change.

Table 8. Transformed sizes of spherical shells for rectangular containers

No.	Inside container size ($L \times W \times H \times \Delta x : mm$)	Parameters of spherical shell		
		Inside radius $r_i (m)$	Outside radius r_o (m)	Thickness Δx (m)
4	298*213*280*26	0.1619	0.1879	0.026
11	170*150*196*56	0.1061	0.1621	0.056
12	230*155*192*40	0.1178	0.1578	0.040

Table 9 shows the duration of package from experiments, the calculated duration of package from the above -- presented packaging models, and their errors.

Table 9. Comparative data on experiment and single-wall models

No.	Container No.	Ice amount(g)	Ambient temperature($°C$)	Real time($hours$)	Calculated times for different models and their errors					
					Model1	error	Model2	error	Model3	error
1	4	8888.6	54.5	30	27.90	-7.01%	34.13	13.78%	29.11	-2.98%
2	11	2012.6	55.0	22.50	23.85	5.98%	25.73	14.34%	24.66	9.61%
3	11	2391.6	55.2	29.25	28.23	3.47%	30.46	4.14%	29.20	-0.17%
4	11	2350.3	58.0	28.33	26.41	-6.79%	28.49	0.56%	27.30	-3.65%
5	12	2959.4	50.3	24.50	24.04	-1.88%	27.23	11.14%	25.46	3.93%
6	12	3055.8	57.8	23.6	21.60	-8.47%	24.47	3.69%	22.83	-3.26%
7	12	3660.9	57.0	28	26.24	-8.99%	29.72	6.16%	27.74	-0.92%
8	12	3503.6	51.0	29	28.07	-3.21%	31.79	9.62%	29.72	2.48%

From Table 9, when convection and radiation are considered, the single-wall spherical shell model matches very well with real insulating packages which only use the insulating container. In fact, this kind of insulating package is popularly used for medical product transportation.

5.1.2 Multi-wall models

Some insulating package system directly uses insulating single-wall container to protect the product. Some time, a corrugated box is used as an outside box to protect the insulating container. In this case there is a thinner air gap between the insulated container and the corrugated box (see Figure 41). To simplify the calculation, the thermal resistance of the package system, R, is used.

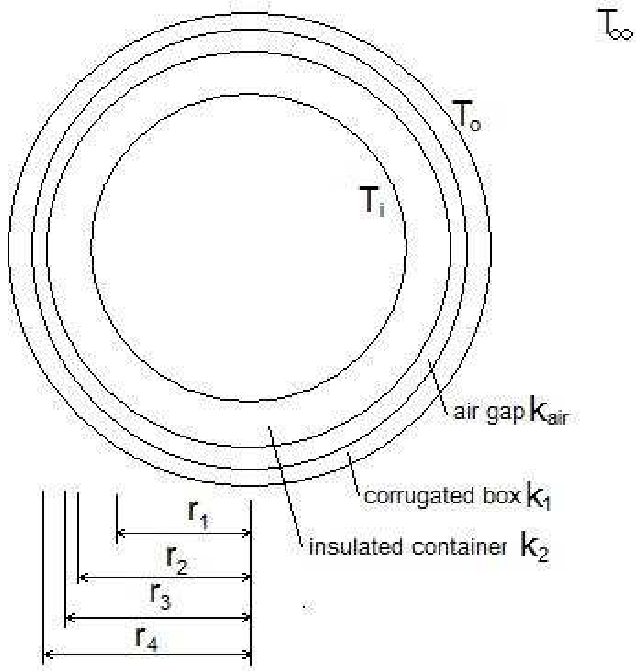


Figure 41. Multi-wall insulating package

$$R = \frac{r_2 - r_1}{4\pi r_1 r_2 k_1} + \frac{r_4 - r_3}{4\pi r_3 r_4 k_2} + \frac{r_3 - r_2}{4\pi r_2 r_3 k_{air}} \tag{5-3}$$

where r_1, r_2, r_3, r_4 in different package systems are shown in Table 10, k_1, k_2, k_{air} are thermal conductivities of corrugated board, insulating material, and air, respectively. The value of thermal conductivities for different insulated materials (containers) could be found on Table 1 and Table 2.

$$k_1 = 0.061 w/m \cdot k, \quad k_{air} = 0.026 w/m \cdot k$$

The dimension of outside corrugated boxes, transformed radius r_1, r_2, r_3, r_4 of the spherical shell and thermal resistance of insulating package system are shown in Table 10.

Table 10. Parameters of insulating package system

System No.	corrugated box size ($L\times W\times H\times \Delta x$: mm)	r_1 (m)	r_2 (m)	r_3 (m)	r_4 (m)	Thermal resistance of system (k/w)
4	375x275x385x3.5	0.2032	0.1997	0.1879	0.1619	2.9132
11	290x278x330x3.5	0.1852	0.1817	0.1621	0.1061	9.2751
12	325x247x268x3.5	0.1725	0.1690	0.1578	0.1178	6.0701

1. Multi-wall packaging model considering only conduction:

The duration of package system is given as:

$$t = \frac{m \cdot \Delta H}{R \cdot \Delta T} \tag{5-4}$$

2. Multi-wall packaging model, considering conduction and convection: According to equation 4-2and 4-7, the following equations can be obtained.

$$h_a \cdot A_1 \cdot (T_\infty - T_o) = \frac{T_o - T_i}{R} = \frac{m \cdot \Delta H}{t}$$

where $A_1 = 4\pi r_1^{\ 2}$.

The surface temperature of insulating package system can be obtained by looking at the first equal sign in the above equation, thus:

$$T_o = \frac{h_a \cdot A_1 \cdot R \cdot T_\infty + T_i}{h_a \cdot A_1 \cdot R + 1}$$

Therefore, the duration of packaging system is determined as:

$$t = \frac{m \cdot \Delta H \cdot R}{T_o - T_i} \tag{5-5}$$

3. Multi-wall packaging model considering conduction, convection and radiation

   In this case, the heat transfer balance can be expressed as following:

$$h_a \cdot A_1 \cdot (T_\infty - T_o) + \varepsilon \sigma A_1 ({T_\infty}^4 - {T_o}^4) = \frac{T_o - T_i}{R} = \frac{m \cdot \Delta H}{t}$$

From the second equal sign of the above equation, the outside temperature of the packaging system can be cast as follows:

$$T_o = \frac{m \cdot \Delta H \cdot R}{t} + T_i$$

Therefore, the equation for duration of insulating package system is also a quadratic equation. Commercial mathematic software (Matlab) is employed to derive the solution of the equation below:

$$h_a \cdot A_1 \cdot (T_\infty - \frac{m \cdot \Delta H \cdot R}{t} - T_i) + \varepsilon \sigma \cdot A_1 \left[{T_\infty}^4 - (\frac{m \cdot \Delta H \cdot R}{t} + T_i)^4 \right] = \frac{m \cdot \Delta H}{t} \tag{5-6}$$

Table 11. Comparative data on experiment and multi-wall models

No.	System No.	Ice amount(g)	Ambient temperature($°C$)	Real time($hours$)	Calculated times for different models and their errors					
					Model1	error	Model2	error	Model3	error
1	4	9137.0	57	34	43.46	27.81%	51.76	52.25%	32.96	-3.68%
2	4	8455.1	57	29	40.21	38.66%	47.90	65.17%	30.50	5.17%
3	4	8469.8	51	31	45.02	45.23%	53.63	73%	33.70	8.7%
4	11	1910.6	50.3	25	32.43	29.72%	34.80	39.2%	23	8%
5	11	2307.2	53.8	27.7	36.62	32.2%	39.29	41.84%	26.28	-5.13%
6	11	2663.1	53.7	32.8	42.34	29.09%	45.44	38.54%	30.38	-7.38%
7	11	2602.7	53.7	31.5	41.38	31.37%	44.41	40.98%	29.69	-5.75%
8	12	2943.0	53.8	25.0	30.90	23.6%	34.83	39.32%	22.68	-9.28%
9	12	3384.6	53.7	29.0	35.60	22.76%	40.14	38.41%	26.12	-9.93%
10	12	3394.2	53.7	29.5	35.70	21.02%	40.25	36.44%	26.20	-11.19%

From Table 11, when three heat transfer modes (conduction, convection and radiation) are considered together, the calculated results for the multi-layer spherical model obviously provide the best answer in terms of error.

5.2 The effect of aluminum foil

According to Figure 31 in chapter 4, the emissivity of the material can significantly affect the thickness of the package and the amount of the PCM for a fixed task of duration. In this section, special material (such as aluminum foil) is applied to the insulating package, and the effect of aluminum foil on the performance of the insulating package is discussed. The duration of package is used for making comparison on the performance of a package when the foil has different locations.

5.2.1 Aluminum foil on the outside surface of the package system

When the aluminum foil is located on the outside surface, the configuration of the package is shown in Figure 42.

Starting from equation 5-6 and using the symbols defined in Figure 42, the duration of the package is expressed as following quadratic equation:

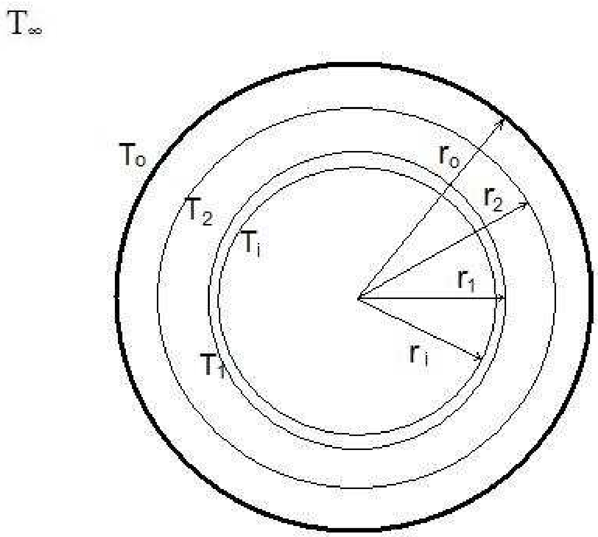


Figure 42. Aluminum foil on the outside

$$h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon\sigma \cdot A_o (T_\infty{}^4 - T_o{}^4) = \frac{4\pi \cdot r_i^3 \cdot \rho \cdot \Delta H}{3t} \quad (5\text{-}7)$$

where $T_o = \frac{4\pi \cdot r_i^3 \cdot \rho \cdot \Delta H \cdot R}{3t} + T_i$ is the outside temperature of insulating package system; $R = \frac{r_o - r_2}{4\pi r_o r_2 k_1} + \frac{r_1 - r_i}{4\pi r_1 r_i k_2} + \frac{r_2 - r_1}{4\pi r_2 r_1 k_{air}}$ is the heat resistance of the package system; t is the duration of the package; h_a is the average natural convection coefficient which is $5.485\, w/m^2 \cdot k$; ρ is the density of PCM; $\varepsilon = 0.07$ is the emissivity of aluminum foil according to Table 3.

5.2.2 Aluminum foil on the outside surface of the inner layer

This configuration is shown in Figure 43. Heat transfer modes of natural convection and radiation at outside surface of the package are considered. Heat transfer mode between r_o and r_2 is conduction; Heat transfer modes of conduction and radiation between r_2 and r_1, that forms two-surface enclosures; Heat transfer mode between r_1 and r_i is conduction.

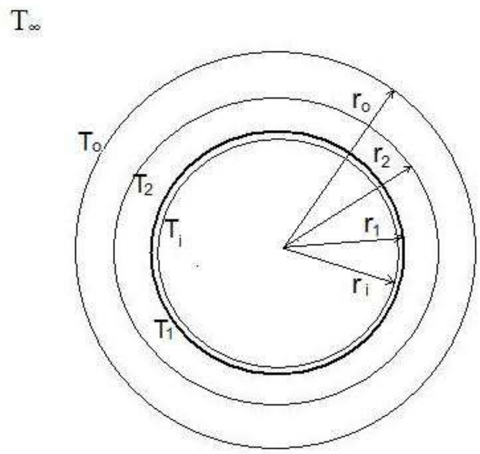


Figure 43. Aluminum foil in the inner

Therefore, basing on section 4.5, the heat transfer equation is expressed as following:

$$
\begin{aligned}
h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon_o \cdot \sigma \cdot (T_\infty{}^4 - T_o{}^4) &= \frac{4\pi \cdot r_o \cdot r_2 \cdot k_1 \cdot (T_o - T_2)}{r_o - r_2} \\
&= \frac{4\pi \cdot r_1 \cdot r_2 \cdot k_a \cdot (T_2 - T_1)}{r_2 - r_1} + \frac{A_1 \cdot \sigma \cdot (T_2{}^4 - T_1{}^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot (\frac{r_1}{r_2})^2} \\
&= \frac{4\pi \cdot r_1 \cdot r_i \cdot k_2 \cdot (T_1 - T_i)}{r_1 - r_i} \\
&= \frac{4\pi \cdot r_i{}^3 \cdot \rho \cdot \Delta H}{3t}
\end{aligned} \tag{5-8}
$$

where ε_o and ε_1 are the emissivity of the outside layer materials; ε_2 is the emissivity of the aluminum foil; k_1, k_2 and k_a are thermal conductivities of the outside and inside layer material and air; ρ is the density of PCM; t is the duration of package; ΔH is the latent heat of PCM.

Using equation 5-8, T_1 and T_2 can be expressed by T_o as follows:

$$T_1 = T_i + \frac{\left[h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon_o \cdot \sigma \cdot A_o \cdot (T_\infty{}^4 - T_o{}^4)\right] \cdot (r_1 - r_i)}{4\pi \cdot r_1 \cdot r_i \cdot k_2}$$

$$T_2 = T_o - \frac{\left[h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon_o \cdot \sigma \cdot A_o \cdot (T_\infty{}^4 - T_o{}^4)\right] \cdot (r_o - r_2)}{4\pi \cdot r_o \cdot r_2 \cdot k_1}$$

Using these two equations to equation 5-8, the equation for T_o can be obtained as follows:

$$h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon_o \cdot \sigma \cdot (T_\infty{}^4 - T_o{}^4)$$

$$= \frac{4\pi r_1 r_2 k_a \left(\left\{ T_o - \frac{\left[h_a A_o (T_\infty - T_o) + \varepsilon_o \sigma A_o (T_\infty{}^4 - T_o{}^4) \right] (r_o - r_2)}{4\pi \cdot r_o \cdot r_2 \cdot k_1} \right\} - \left\{ T_i + \frac{\left[h_a A_o (T_\infty - T_o) + \varepsilon_o \sigma A_o (T_\infty{}^4 - T_o{}^4) \right] (r_1 - r_i)}{4\pi \cdot r_1 \cdot r_i \cdot k_2} \right\} \right)}{r_2 - r_1}$$

$$+ \frac{A_1 \sigma \left(\left\{ T_o - \frac{\left[h_a A_o (T_\infty - T_o) + \varepsilon_o \sigma A_o (T_\infty{}^4 - T_o{}^4) \right] (r_o - r_2)}{4\pi \cdot r_o \cdot r_2 \cdot k_1} \right\}^4 - \left\{ T_i + \frac{\left[h_a A_o (T_\infty - T_o) + \varepsilon_o \sigma A_o (T_\infty{}^4 - T_o{}^4) \right] (r_1 - r_i)}{4\pi \cdot r_1 \cdot r_i \cdot k_2} \right\}^4 \right)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \cdot \left(\frac{r_1}{r_2} \right)^2}$$

Using mathematical software (Matlab) to solve this quadratic equation T_o can be found out. Substituting T_o into the below equation, the duration of the package can be obtained:

$$t = \frac{4\pi \cdot r_i{}^3 \cdot \rho \cdot \Delta H}{3\left[h_a \cdot A_o \cdot (T_\infty - T_o) + \varepsilon_o \cdot \sigma \cdot (T_\infty{}^4 - T_o{}^4) \right]} \tag{5-9}$$

5.2.3 Comparative solution

Comparing Figure 42 and Figure 43, it can be clearly seen that the only difference between these two figures is the different locations of the aluminum foil. To further simplify the process, all parameters may be kept as follows:

$k_1 = k_2 = 0.04W / m \cdot K$; $k_a = 0.026W / m \cdot K$;
$\varepsilon = \varepsilon_1 = 0.07$; $\varepsilon_o = \varepsilon_2 = 0.9$;
$T_\infty = 330K$; $T_i = 273K$;
$\rho = 900Kg / m^3$; $\Delta H = 335000J / Kg$.

Using the previous models, the location effect of the aluminum foil can be investigated. Alternating the emissivity at these two different locations, the corresponding changes of duration of the insulating package are shown in Table 12. The geometric dimensions of the insulating package system I as follows.

$r_o = 0.2m$; $r_2 = 0.16m$; $r_1 = 0.15m$; $r_i = 0.145m$.

The geometric dimensions of the insulating package system II as follows.

$r_o = 0.4m$; $r_2 = 0.36m$; $r_1 = 0.35m$; $r_i = 0.345m$.

From Table 12, it can be concluded that, when aluminum foil is applied on the outside of the package system, the duration is approximately 4% longer. In another words, applying the aluminum foil on the outside surface is more effective than applying it on an inner layer.

Table 12. Duration of the insulating package system with different Al foil location

	Al foil on outside	Al foil in inner
System I	82.27 hours	79.27 hours
System II	232.15 hours	224.35 hours

CHAPTER 6. INSULATING PACKAGING SOLUTIONS

In chapters 4 and 5, various insulating package models have been presented and discussed, and validated by experiment. It can be concluded that the transformation of a rectangular container to a spherical shell container is a reliable and effective modeling tool. The spherical shell model can be applied to predict the effectiveness of an insulating package. But this is not the final purpose of this model. As mentioned previously, the purpose in developing this spherical shell model is for designing the insulating package.

From Chapter 4, when the demands of a package (such as ambient temperature, shipping time and the size of product) are determined, the designer can apply the relationship curve of $r_i - \Delta x$ (inside radius-thickness on spherical shell) for a set of given inputs. In this case the designer faces many options for inside radiuses and thicknesses. Each combination offers different package design solutions while can satisfy all the basic given requirements. However, the designer wants to determine a single optional solution in terms of a set of specifics. Here 'the minimum cost' rule is used as the optimized target. Lowest cost is the ultimate goal of the designer when the package can meet all of packaging functions. To simplify the explanation of the package solution, the single-wall spherical shell model is discussed in this section, with minimum packaging cost as an optimizing target for the insulating package design.

To reduce the cost of packaging is to reduce the cost of the package itself. In insulating packages, the packaging cost includes the cost of the packaging container and the PCM. In this case, the cost of the package can be expressed as follows:

$$\begin{aligned} Cost &= Cost \quad of \quad Container \quad + \quad Cost \quad of \quad PCM \\ &= Spherical \quad Shell \quad Mass \quad \times A \quad + \quad PCM\ Mass \quad \times \quad B \\ &= \frac{4}{3}\pi\rho_1(r_o{}^3 - r_i{}^3)\cdot A + \frac{4}{3}\pi\rho_2(r_i{}^3 - r_p{}^3)\cdot B \end{aligned} \tag{6-1}$$

where r_o , r_i and r_p are the outside radius, inside radius of spherical shell and the radius of the product, respectively. A is the cost of insulated material per unit mass. B is the cost of PCM per unit mass. ρ_1 and ρ_2 are the density of the insulated material and PCM, respectively.

Therefore, to have an insulating package design, the procedure is:

1. Start from the curve of the inside radius-thickness of spherical shell ($r_i - \Delta x$). Equation 4-13 provides the relationship between the inside radius r_i and the thickness of spherical shell Δx when both the outside convection and radiation are considered:

$$\begin{aligned} &3t\cdot h_a\cdot(r_i+\Delta x)^2(T_\infty - T_i - \frac{\rho\cdot(r_i^3 - r_p{}^3)\cdot\Delta H\cdot\Delta x}{3t\cdot(r_i+\Delta x)\cdot r_i\cdot k}) \\ &\quad + 3t\cdot\varepsilon\cdot\sigma\cdot(r_i+\Delta x)^2[T_\infty{}^4 - (T_i + \frac{\rho\cdot(r_i^3 - r_p{}^3)\cdot\Delta H\cdot\Delta x}{3t\cdot(r_i+\Delta x)\cdot r_i\cdot k})^4] \\ &\qquad = \rho\cdot(r_i^3 - r_p{}^3)\cdot\Delta H \end{aligned} \tag{4-13}$$

2. Determine the related parameters from the inputs in equation (4-13). To shorten the experiment time, assume the duration of the package is 24 hours. All parameters in the equation are shown as following.

$t = 24hours$; $h_a = 5.485w/m^2\cdot°c$; $T_\infty = 50°C$; $\rho = 600Kg/m^3$;
$\Delta H = 335000KJ/Kg$; $k = 0.039w/m\cdot°C$; $\varepsilon = 0.9$ $r_p = 0.075m$

4. Generate the curve of the inside radius-thickness of spherical shell ($r_i - \Delta x$). Figure 44 shows the relationship between the inside radius and the thickness of the spherical shell when the combination convection and radiation is considered

for the above given set of inputs.

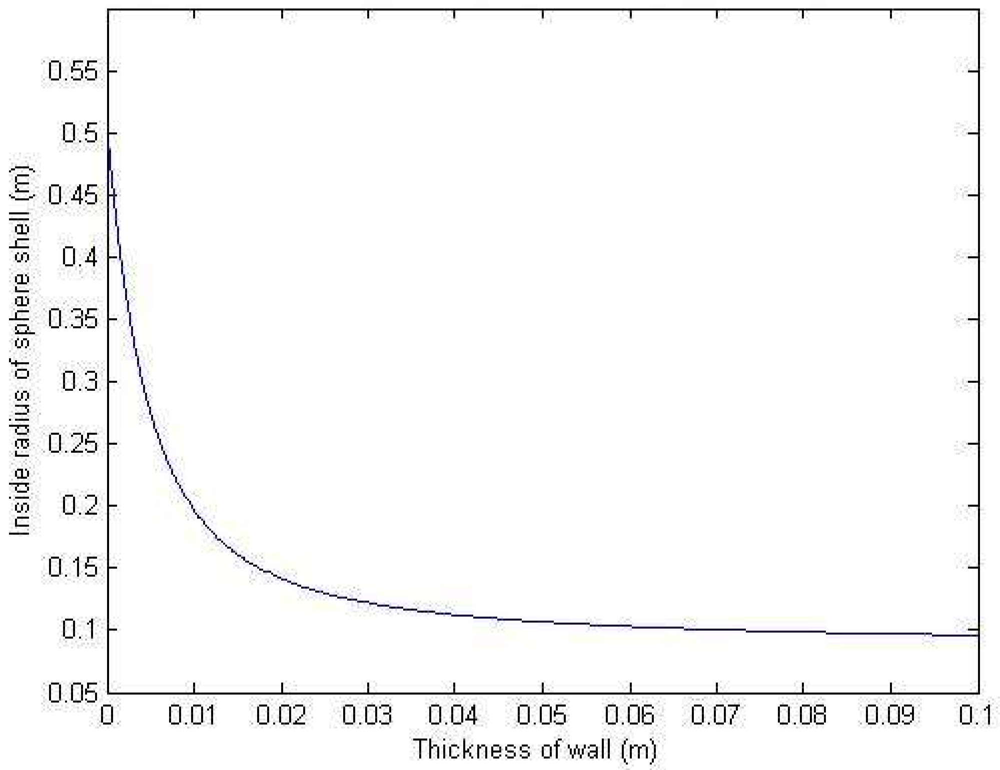


Figure 44. Inside radius of shell – thickness of wall curve on package solution

5. Determine the design. After setting the optimizing target as the minimum cost, an additional relation can be created to determine the design condition on the curve. According to the reference price for insulated containers and "ice-brix" cold packs on the website (http://www.fast-pack.com/insulated-shipping-containers.html), the price of containers is \$6/kg and the price of ice gel pack is \$0.2/kg. Applying all the data to Figure 44, one can derive the cost-thickness curve based on equation 6-1 as shown in Figure 45.

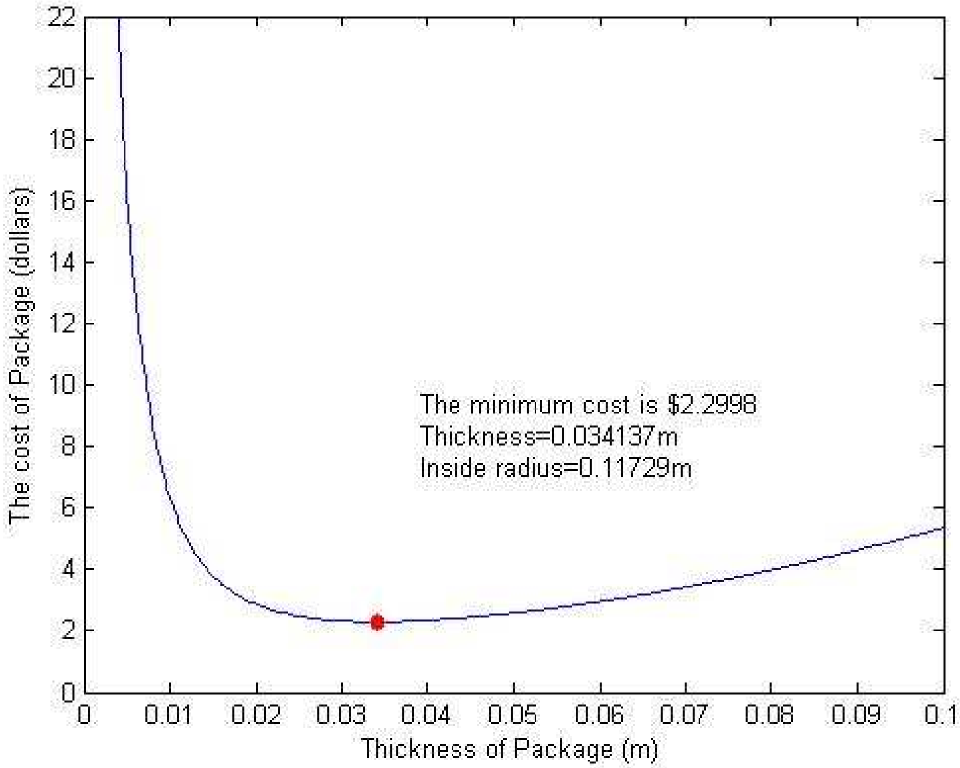


Figure 45. Cost-thickness curve

Figure 45 shows that the minimum cost of package is \$2.30 when the thickness of package is 34.1 mm and the inside radius of spherical shell is 117.3 mm from the given cost information of the materials. When $r_i = 0.1173m$; $r_p = 0.075m$, the quantity of ice can be calculated using this equation:

$$\frac{4}{3}\pi \cdot (0.11723^3 - 0.075^3) \cdot 600000$$
$$= 2988.8g$$

Therefore, not only is the design of the package complete, but the cost and the quantities of materials become available to users.

4. Return the spherical shell design to rectangular container design. It is well known that a spherical shell is not practical for packaging applications. The spherical shell model only serves as an analog for the purpose of packaging design. Designers should be able to transform the spherical shell back to rectangular container. Begin with a cubic container with inside dimensions a. Based on the transforming principle

developed in this research, the side a can be calculated by the following equation, equating the inner shell volume to the interior volume of the cube,

$$a = (\frac{4}{3}\pi r_i^3)^{1/3} \tag{6-2}$$

Applying the optimized result of the inner shell radius, the inside side of sample cubic is:

$$\begin{aligned} a &= (\frac{4}{3}\pi r_i^3)^{1/3} \\ &= (\frac{4}{3}\pi \cdot 0.11723^3)^{1/3} \\ &= 0.1891m \end{aligned}$$

To verify this optimized result, an experiment needs to be set up for the final container. Unfortunately, an exact dimension of the cubic container as given above is not available due to resource limitations. The radius of available box No. 12 (230x155x192x40mm) is 0.1178 m when it is transformed to the spherical shell. It exactly matches with the optimized result. But the thickness of box No. 12 is 40 mm, which is thicker than the calculated result (34.14mm). Using Figure 44, when the thickness of container is 0.04m, the inside radius of spherical shell is 0.1134m and the quantity of broken ice is 2605 g. Box No. 12 is still a good choice because a thicker box can be understood as a safe factor for design. Therefore box No.12 is chosen to be tested to verify the optimized design.

5. Verify package solution. A verification test is run on the chosen rectangular insulating container (box No. 12), loading the calculated amount of PCM (2988.8 g broken ice) into the container as a packaging prototype, setting up the experiment with ambient temperature, and obtaining the test data. The duration of the packaging system is selected to verify the modeling of the developed spherical shell model.

Figure 46 shows the ambient temperature during the experiment. It can be seen that the ambient temperature is kept at 50 ℃.

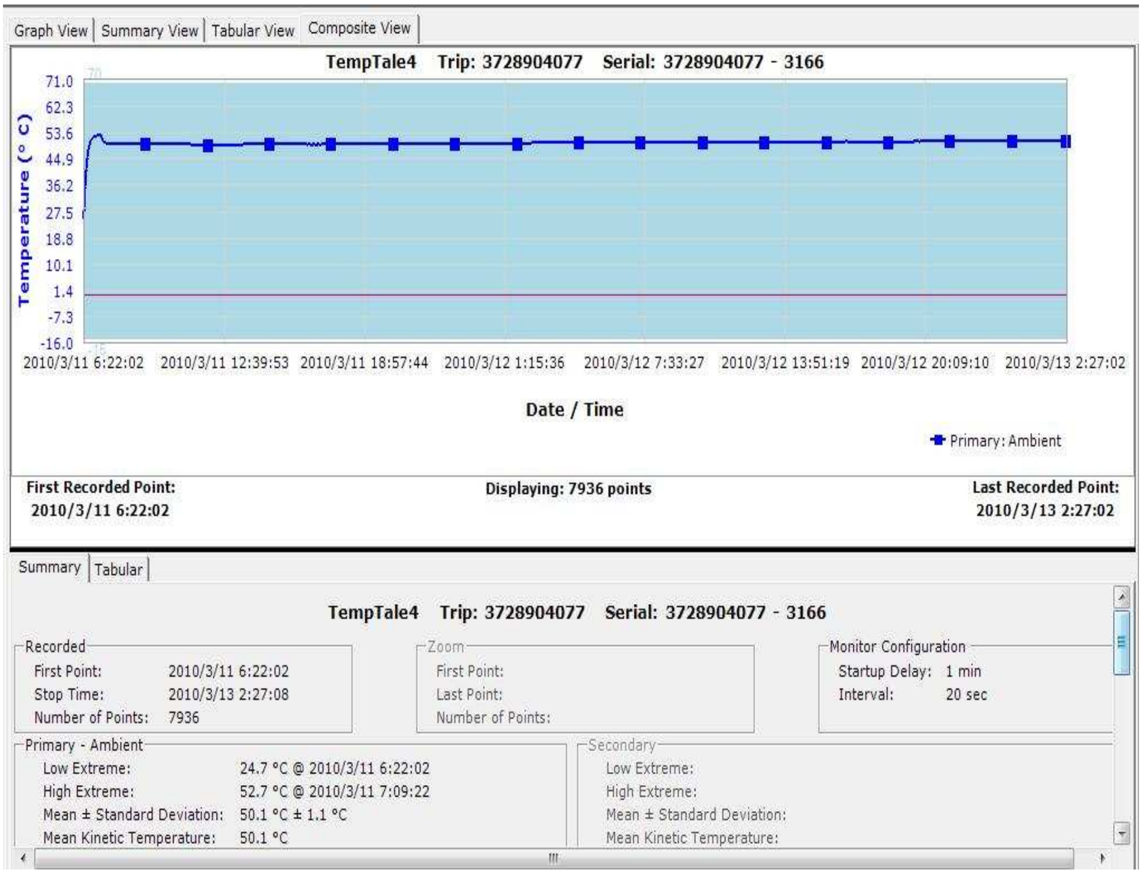


Figure 46. Ambient temperature on validation experiment

Figure 47 recorded inside temperature of package during experiment.

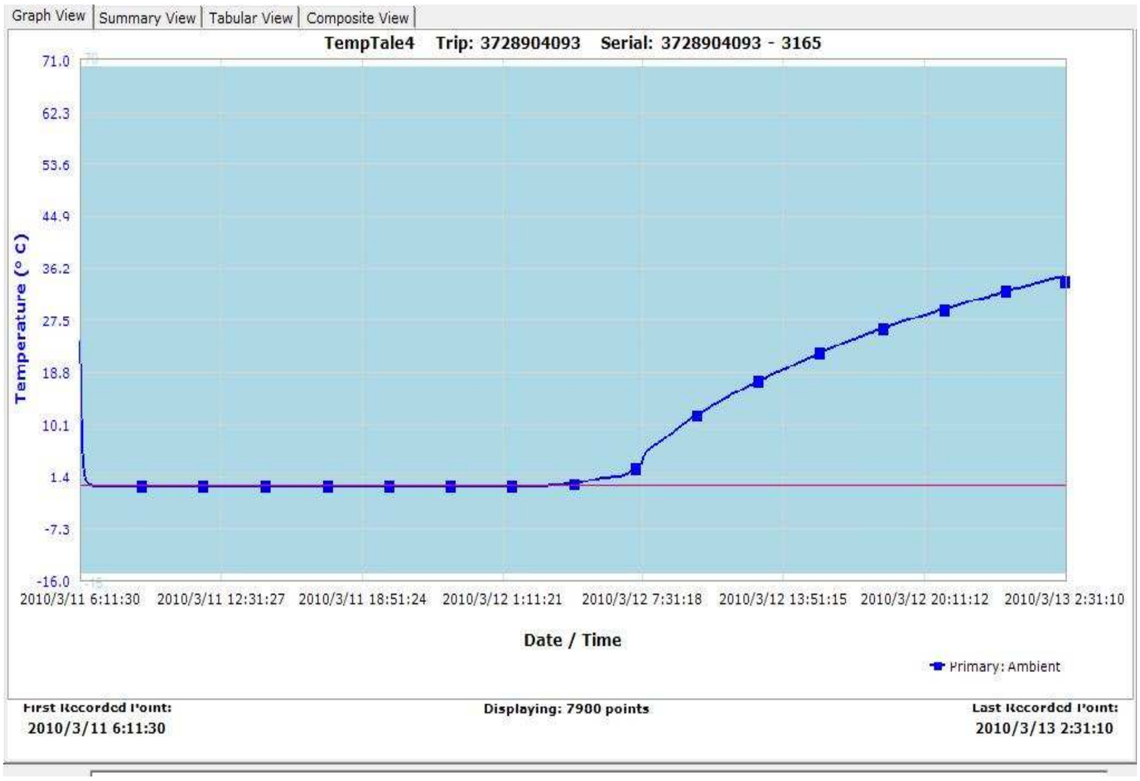


Figure 47. Inside temperature on validation experiment

Figure 47 shows the starting time of experiment was 6:11, and ending time was 7:16 of the next day, when all the ice has melted. In this case the duration of the package is 25 hours, 5 minutes. Since the thickness of the rectangular container is greater than in the spherical shell model, the duration of the package in the experiment is 65 minutes longer. This shows that the insulating package design is successful.

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

In this dissertation, a spherical shell model has been developed to simplify heat transfer analysis and the design of an insulating package. This spherical shell model provides a working foundation for such a package design by transforming a rectangular insulated container to a spherical shell configuration, in which the spherical shell model with the inside volume of the spherical shell equaling to the inside volume of the container and the thickness of spherical shell equaling to the thickness of container. The closed form of the mathematical formulation for the various heat transfer modes through a spherical shell allows an analysis and optimization in the applications of insulating package design. The optimized package design under all given inputs is then transformed back to a rectangular configuration.

Applying the 'minimum cost' principle, designer can decide the size of insulated container, the amount of PCM and the configuration of the packaging system from the general formulation of the solution of spherical shell model. After this prototype packaging design is completed, a benchmark test can be set up and conducted to verify this theoretical solution.

The experimental data well supports the spherical shell model developed in this research.

However, this spherical shell model is limited in application to cubic configurations for package containers. A departure from a cubic geometry to a rectangular configuration will create errors. To confine or extend the current approach to more generic applications, the following recommendations are made.

1. An appropriate aspect ratio range of rectangular containers should be determined to make sure that the spherical shell model has reasonable

calculated accuracy.

2. For those rectangular containers which are not suitable to apply the spherical shell model, a new model should be developed.
3. For further research work, the temperature variation during the phase change of PCMs should be considered.

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